



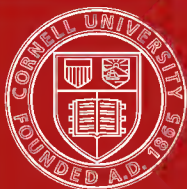
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**The temperature-entropy diagram,**



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**W. N. BARNARD.**



THE  
TEMPERATURE-ENTROPY  
DIAGRAM

BY  
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THIRD EDITION, REVISED AND ENLARGED.

FIRST THOUSAND

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BY

CHARLES W. BERRY

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## PREFACE TO THIRD EDITION

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THE present revision includes the following additions and changes: Minor insertions have been made in the chapters upon the Flow of Fluids, the Gas Engine Cycles, and the Non-conducting Steam Engine. The chapter on Refrigeration and the Warming Engine has been expanded into separate chapters upon each subject. A special chapter has been added upon Entropy Analysis in the Boiler Room. The Tables upon the Efficiency, Water and Heat Consumption of the Rankine Cycle have been extended to cover the range of low-pressure turbines as well as high-pressure reciprocating engines. All illustrative problems have been recalculated to agree with the most recent and accurate data upon steam. The second and third editions of this book have so extended its scope that it is now a treatise upon graphical thermodynamics although still abiding by the limitations imposed by its title.

CHARLES W. BERRY.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,  
January, 1911.

## PREFACE TO SECOND EDITION.

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IN the revised edition of the Temperature-Entropy Diagram a more extended application of the principles of the  $T\phi$ -analysis to advanced problems of thermodynamics has been made than was possible in the limited scope of the previous edition. The Chapter on the Flow of Fluids has been entirely rewritten and treats at length various irreversible processes. A graphical method of projecting from the  $pv$ - into the  $T\phi$ -plane has been elaborated for perfect gases and its application illustrated in the chapters on Hot-air Engines and Gas-engines. The various factors affecting the cylinder efficiency of both gas- and steam-engines have been thoroughly discussed. One chapter has been devoted to the thermodynamics of mixtures of gases and vapors, and another to the description and use of Mollier's total energy-entropy diagram.

CHARLES W. BERRY.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,

October, 1908.

## PREFACE TO THE FIRST EDITION.

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THIS little volume was prepared for the use of students of thermodynamics, and therefore I have endeavored to bring together in logical order certain information concerning the construction, interpretation, and applications to engineering problems of the temperature-entropy diagram, which otherwise would not be readily available for them, as such information is scattered throughout various treatises. The book is not intended for the advanced student, as he is already familiar with its contents, neither is it expected that one entirely ignorant of thermodynamics can use it to advantage, as the reader is assumed to have an elementary knowledge of the fundamental theory and equations. An exhaustive treatment has not been attempted, but it is believed that the graphical presentation here given will aid the student to a clearer comprehension of the fundamental principles of thermodynamics and make it possible for him to read understandingly more pretentious works.

CHARLES W. BERRY.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,  
January, 1905.



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## INTRODUCTION.

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It seems necessary in a book dealing with the application of the temperature-entropy diagram to discuss briefly that "ghostly quantity," entropy, although I do not intend to give any new definition of a function already too variously defined, but rather to pick out such of the present ones as are correct.

One has but to plot an irreversible adiabatic process in the temperature-entropy plane to realize once and for all that the entropy does not necessarily remain constant along an adiabatic line. In fact isentropic and adiabatic changes coincide only when the latter process is reversible: and such a change practically never occurs in nature. For example, in one irreversible adiabatic expansion representing the flow through a non-conducting porous plug, the heat added is zero, so that  $\int \frac{dQ}{T} = 0$ , but nevertheless the entropy of the substance increases. It is even possible to imagine an irreversible process which is at the same time isentropic. Suppose a gas to expand through a nozzle

losing heat by radiation and conduction and also undergoing friction losses whereby part of its kinetic energy is dissipated and restored to the gas as heat. The loss of heat by radiation and conduction will reduce the entropy of the gas, while the gain of heat by friction will increase it. It is possible to consider these two opposing influences as equal, and then the flow will be isentropic although not adiabatic.

The entropy of a substance, just as much as its intrinsic energy, specific volume, specific pressure, or temperature, has a definite value for each position of the state point upon the characteristic surface, and the increase in the value of the entropy in changing from one point to another is a definite quantity regardless of the path chosen. The magnitude of this increase is equal to  $\int_1^2 \frac{dQ}{T}$ , taken along any *reversible* path between these points. This fact has led to the inexact definition of change of entropy,  $d\phi = \frac{dQ}{T}$ , a definition true only for ideal reversible processes and hence utterly wrong when applied to actual irreversible processes, as in general  $d\phi > \frac{dQ}{T}$ .

Since for reversible cycles  $d\phi = \frac{dQ}{T}$ , it follows that the heat added during any reversible change is equal to  $Q = \int_1^2 T d\phi$ , and for an isothermal process

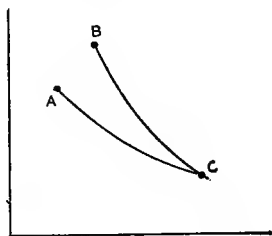


$Q = T(\phi_2 - \phi_1)$ . This is undoubtedly the basis for all the physical analogies attempting to explain entropy as heat-weight, etc., and also for the name "heat diagram" applied to the temperature-entropy diagram. The area under a curve in the  $T\phi$ -plane is equal to the heat received from, or rejected to, some outside body only when the process is reversible.

Similarly if the specific pressure and specific volume of a gas could be ascertained at various points in its passage through a porous plug, these points if plotted would form a  $pv$ -curve giving a true history of the movement of the state point, but the area under the curve would not represent work, as no external work has been performed.

Preston in his *Theory of Heat* says: "The entropy of a body being taken arbitrarily as zero in some standard condition  $A$ , defined by some standard temperature and pressure (or volume), the entropy in any other state  $B$  is the value of  $\int \frac{dQ}{T}$  taken along any reversible path by which the body may be brought to  $B$  from the standard state  $A$ . The path may obviously be an arc  $AC$  of an isothermal line passing through the point defining the standard state, together with the arc  $BC$  of the adiabatic line passing through  $B$ . The entropy in the state  $B$  may consequently be measured thus. Let the volume be changed adiabatically" (reversible process) "until the standard temperature  $T$  is attained,

and then change the volume isothermally until the standard pressure is attained. If the quantity of heat



imparted during the latter operation be  $Q$ , the entropy in the state  $B$  is  $\phi = \frac{Q}{T}$

“In this operation the temperature and pressure are supposed uniform throughout the body. . . . If, however, any body be subject to operations which produce inequalities of temperature in the mass, there will be a transference of heat from the warmer to the colder parts by conduction and radiation, and although the body may neither receive heat from nor give it out to other bodies (so that the transformation is adiabatic throughout), yet on account of the inequalities of temperature, the entropy of the mass will increase, . . . and under these circumstances the transformation will not be isentropic.”

Swinburne in his *Entropy* says: “Entropy may be defined thus: Increase of entropy is a quantity which, when multiplied by the lowest available temperature,

gives the incurred waste. In other words, the increase of entropy multiplied by the lowest temperature available gives the energy that either has been already irrevocably degraded into heat during the change in question, or must, at least, be degraded into heat in bringing the working substance back to the standard state. . . .

“Thus the entropy of the body in state  $B$  is not a function of the heat actually taken in during its change from  $A$  to  $B$ , as the change must have been partially, and may have been wholly, irreversible; but it can be measured as a function of the heat which would have to be taken in to change from  $A$  to  $B$  reversibly, or which would have to be given out if the substance were changed from  $B$  to  $A$  reversibly, which amounts to the same thing. . . .

“The entropy of a body therefore depends only on the state, and not on its past history.”

Planck in his *Treatise on Thermodynamics* writes (see English translation by Ogg):

“A process which can in no way be completely reversed is termed *irreversible*, all other processes *reversible*. That a process may be irreversible, it is not sufficient that it cannot be directly reversed. This is the case with many mechanical processes which are not irreversible. The full requirement is, that it be impossible, even with the assistance of all agents in nature, to restore everywhere the exact initial state when the process has once taken place. . . . The gen-

eration of heat by friction, the expansion of a gas without the performance of external work, and the absorption of external heat, the conduction of heat, etc., are irreversible processes.

“Since there exists in nature no process entirely free from friction or heat-conduction, all processes which actually take place in nature, if the second law be correct, are in reality irreversible; reversible processes form only an ideal limiting case. They are, however, of considerable importance for theoretical demonstration and for application to states of equilibrium.

“If a homogeneous body be taken through a series of states of equilibrium, that follow continuously from one another, back to its initial state, then the summation of the differential  $\frac{dE + pdv}{T}$  extending over all the states of that process gives the value zero. It follows that, if the process be not continued until the initial state, 1, is again reached, but be stopped at a certain state, 2, the value of the summation  $\int_1^2 \frac{dE + pdv}{T}$  depends only on the states 1 and 2, not on the manner of the transformation from state 1 to state 2. . . .

“The (above) integral . . . has been called by Clausius the entropy of the body in state 2, referred to state 1 as the zero state. The entropy of a body in a given state, like the internal energy, is completely

determined up to an additive constant, whose value depends on the zero state.

*"It is impossible in any way to diminish the entropy of a system of bodies without thereby leaving behind changes in other bodies.* If, therefore, a system of bodies has changed its state in a physical or chemical way, without leaving any change in bodies not belonging to the system, then the entropy in the final state is greater than, or, in the limit, equal to the entropy in the initial state. The limiting case corresponds to reversible, all others to irreversible, processes.

"The restriction . . . that no changes must remain in bodies outside the system is easily dispensed with by including in the system all bodies that may be affected in any way by the process considered. The proposition then becomes:

*"Every physical or chemical process in nature takes place in such a way as to increase the sum of the entropies of all the bodies taking any part in the process. In the limit, i.e. for reversible processes, the sum of the entropies remains unchanged. This is the most general statement of the second law of Thermodynamics."*



## SYMBOLS USED IN THE FOLLOWING PAGES.

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$A = \frac{1}{J}$  = Heat equivalent of a unit of work.

$Apu$  = Heat equivalent of the external work of vaporization.

$c$  = General expression for the specific heat during any change.

$c_p$  = Specific heat at constant pressure.

$c_v$  = Specific heat at constant volume.

$\Delta$  = Change of . . .

$E$  = Internal (intrinsic) energy in work units =  $S + I$ .

$\eta$  = Thermal efficiency of an engine.

$F$  = Area (*Flache*).

$G$  = Weight (*Gewicht*).

$g$  = Acceleration due to gravity.

$H$  = Total heat above some arbitrary zero, =  $q$ ,  $q + xr$ ,  $\lambda$ ,  
 $\lambda + c_p(t_s - t)$ .

$h$  = Specific heat of dry saturated vapor.

$I$  = Internal energy due to separation of molecules.

$i$  = Total energy =  $A(E + pv) = H + Ap\sigma$ .

$k = \frac{c_p}{c_v}$  for a perfect gas.

$\lambda$  = Total heat of dry saturated vapor =  $q + r$ .

$n$  = Exponent of the polytropic change  $pv^n = c$ .

$p$  = Specific pressure.

$\phi$  = General expression for entropy.

$Q$  = Heat received from or exhausted to some outside body.

$q$  = Heat of the liquid.

$R = \frac{pv}{T}$ , for a perfect gas.

$r$  = Total latent heat of vaporization =  $\rho + Apu$ .

*SYMBOLS USED IN THE FOLLOWING PAGES.*

$\rho$  = Internal latent heat of vaporization.

$\frac{r}{T}$  = Entropy of vaporization.

$S$  = Internal energy due to vibration of molecules.

$s$  = Specific volume of a dry saturated vapor.

$\sigma$  = Specific volume of a liquid.

$T$  = Temperature in degrees absolute, Fahrenheit or Centigrade.

$t$  = Temperature in degrees Fahrenheit or Centigrade.

$t_s$  = Temperature of superheated vapor.

$\theta$  = Entropy of the liquid =  $\int \frac{cdt}{T}$ .

$u$  = Increase of volume due to vaporization =  $s - \sigma$ .

$v$  = General value of specific volume =  $\sigma$ ,  $xu + \sigma$ ,  $s$ , etc.

$V$  = Velocity in linear units per second.

$\frac{V^2}{2g}$  = Kinetic energy of a jet in work units.

$W$  = Work performed by or upon a substance.

$x$  = Quality of a unit weight of a mixture of a liquid and its vapor.



# THE TEMPERATURE-ENTROPY DIAGRAM.

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## CHAPTER I.

### GENERAL DISCUSSION. REVERSIBLE PROCESSES AND CYCLES. EFFECT OF IRREVERSIBILITY.

THE condition of a substance is in general completely defined by any two of its five characteristic properties, specific pressure, specific volume, absolute temperature, intrinsic energy, and entropy. The relations existing between any three of these qualities being expressed by the formula  $z=f(x, y)$ . Exceptions to this occur in the case of saturated vapors, where specific pressure and temperature alone do not suffice to define completely the state of the body, and in the case of perfect gases, where isodynamic and isothermal changes coincide.

In the analytical solution of thermodynamic problems those formulæ are used which contain the properties most important for the investigation in hand. Similarly in graphical solutions any pair of character-

istics may be used as coordinates as convenient and then the curves,  $y=f(x)$ , expressing the relations between the various characteristics drawn in this  $xy$ -plane, assume different forms according to the laws of variation of the five variables  $p, v, T, E$ , and  $\phi$ .

The pressure-volume diagram, or  $pv$ -diagram, is the one most widely used, as its coordinates are those common to every-day experience, but for some investigations of heat-transference, changes of temperature, changes of entropy, etc., the temperature-entropy diagram, or the  $T\phi$ -diagram, lends more ready assistance.

As a consequence of the fundamental relation  $z=f(x, y)$ , any curve  $p=f_1(v)$  in the  $pv$ -plane has its counterpart  $\phi=f_2(T)$  in the  $T\phi$ -plane; both being but special projections of the same change on the characteristic surface.

In the following a discussion will be given of the different forms assumed by the curve  $\phi=f(T)$ , in the case of perfect gases, saturated steam, and superheated steam under the conditions of constant temperature, constant entropy, constant pressure, constant volume, constant intrinsic energy, etc., and also a consideration of the interpretation to be given to the areas included between any given curve and the axis, together with certain other interesting details.

If  $O\phi$  and  $OT$  (Fig. 1) represent the axis of entropy and temperature respectively, any isothermal change,

as at the temperature  $T_2$ , will be represented by a horizontal line  $ab$ , and similarly any isentropic change (or adiabatic in the case of a reversible process) will be represented by a vertical line, as  $cd$ . The forms of the curves for constant pressure, etc., will vary with

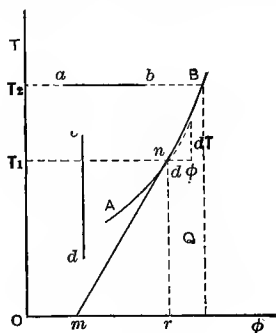


FIG. 1.

the state of the substance and will be investigated for each special case.

Let  $AB$  represent any *reversible* process and let  $c$  be the specific heat of the substance during this change. In order to increase the temperature of a unit weight of the substance by the amount  $dT$  there will be necessary the expenditure of the heat  $dQ = cdT$ . But for a reversible cycle there exists the further relation

$d\phi = \frac{dQ}{T}$ , or  $dQ = Td\phi$ , and therefore

$$\left. \begin{aligned} dQ &= cdT = Td\phi, \\ Q &= \int_{T_1}^{T_2} cdT = \int_{\phi_1}^{\phi_2} Td\phi. \end{aligned} \right\} \dots (1)$$

or

Now  $\int_{\phi_1}^{\phi_2} T d\phi$  represents the area included between the curve  $AB$  and its projection upon the  $\phi$ -axis. Hence *the heat necessary to produce any reversible change is represented by the area under the curve in the  $T\phi$ -plane.*

The differential form of equation (1) leads at once to the expression

$$c = T \frac{d\phi}{dT}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

At any point  $n$  of the curve  $AB$  draw the tangent  $nm$  and construct the infinitesimal triangle  $dt d\phi$ . Then from similar triangles

$$mr : T = d\phi : dT,$$

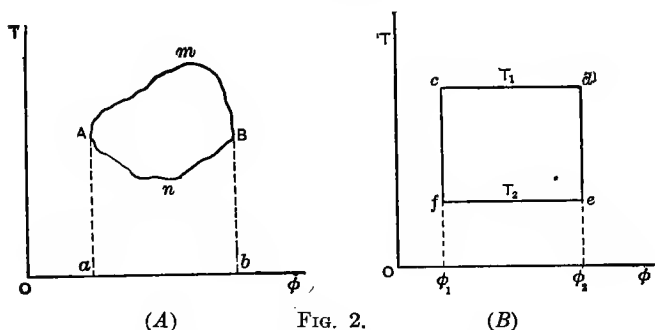
$$\text{or} \quad mr = T \frac{d\phi}{dT} = c;$$

i.e., if from any point in a curve representing a reversible process a tangent be drawn, the length of the subtangent on the  $\phi$ -axis represents the momentary value of the specific heat for that change.

In Fig. 2 (A) let  $AmBnA$  represent a reversible cycle in which  $AmB$  is the forward stroke and the area  $aAmBb$  represents the heat received from external sources, and where  $BnA$  is the return stroke and the area  $aAnBb$  represents the heat rejected. On the completion of the cycle the intrinsic energy has regained its initial value and therefore the difference between

the heat received and the heat rejected, i.e., the magnitude of the enclosed area  $AmBnA$ , must represent the amount of heat changed into work.

**Carnot Cycle.**—In the case of the Carnot engine this choice of coordinates leads to a beautiful simplicity. The cycle Fig. 2 (B) becomes a rectangle consisting



of (1) the isothermal expansion  $cd$ , during which is received the heat  $Q_1 = \int_{\phi_1}^{\phi_2} T_1 d\phi = T_1(\phi_2 - \phi_1)$ , represented by the total rectangle  $\phi_1 cd\phi_2$ ; (2) the adiabatic line  $de$ ; (3) the isothermal compression during which is rejected the heat  $Q_2 = \int_{\phi_1}^{\phi_2} T_2 d\phi = T_2(\phi_2 - \phi_1)$ , represented by  $\phi_1 fe\phi_2$ ; and (4) the adiabatic compression  $fc$ . The heat changed into work is  $Q_1 - Q_2 = T_1(\phi_2 - \phi_1) - T_2(\phi_2 - \phi_1) = (T_1 - T_2)(\phi_2 - \phi_1)$ . The efficiency,  $\eta = \frac{Q_1 - Q_2}{Q_1}$ , is simply the ratio of the rectangles  $cdef$  and  $\phi_1 cd\phi_2$ , and as these have the same base, the

areas are proportional to the altitudes and at once the efficiency becomes  $\eta = \frac{T_1 - T_2}{T_1}$ .

**Isodiabatic Cycles.**—In deducing the equation  $d\phi = \frac{dQ}{T}$  for reversible cycles, use is made of the Carnot cycle and it is shown that no other cycle can have a greater efficiency. There are, however, a number of other cycles having the same efficiency as the Carnot within the same range of temperature.

In Fig. 3, let the reversible cycle  $abcd$  be formed by

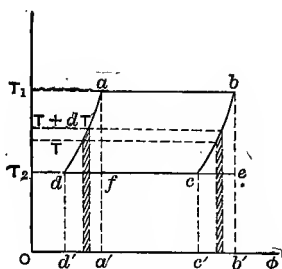


FIG. 3.

the two isothermals  $ab$  and  $cd$  and of the two curves  $bc$  and  $da$ . The curve  $bc$  is arbitrary, but  $da$  is drawn like it, being simply displaced to the left. The heat turned into work during the cycle is  $abcd$ , or equal to that of the equivalent Carnot cycle  $abef$ , but the heat supplied is  $d'daa' + a'abb'$ , or is greater than that needed to perform the same work with the Carnot cycle by the amount  $d'daa'$ . Now the heat rejected from  $b$  to  $d$  consists of the two parts,  $cd d'c'$ , which passes to the

refrigerator at the lowest temperature  $T_2$  and is of necessity lost, and  $bb'c'c$ , which is rejected during a dropping temperature and which is strictly equal to the heat required to carry out the reverse operation  $da$ . Instead of being wasted, the heat rejected along  $bc$  might be stored up and returned to the fluid along  $da$ , thus making no further demands upon the source of heat. In this manner the one operation would balance the 'other and the heat actually required from an external source would be represented by  $a'abb'$ , as in the Carnot cycle. The heat actually rejected would also be represented by  $d'dcc'$ , equal to  $a'feb'$ , as in the Carnot cycle. Thus the *adiabatic* lines of the Carnot cycle would be replaced by two lines  $bc$  and  $da$ , along which the interchanges of heat are compensated. The efficiency of such *isodiabatic* cycles would thus be equal to that of the Carnot cycle.

The operations along the lines  $bc$  and  $da$  may be imagined as follows. Heat only flows spontaneously from one body to another at a lower temperature. Thus the heat given up along  $bc$  can be stored and utilized to effect the operation  $da$ , if the process be subdivided into very small differences of temperature and each portion of the heat rejected at the momentary temperature. Thus the heat rejected along  $bc$  between the temperatures  $T' + dT'$  and  $T'$  (Fig. 3) is returned along a portion of  $da$  at the same temperature.

The difficulty is to find a practical regenerator to

perform this duty. It must have large heat-conducting surfaces, and consist of a number of subdivisions at all temperatures between  $T_1$  and  $T_2$ , and there must be no conduction of heat from any part to the next at lower temperature. To accomplish the operation along  $bc$ , the working fluid passes successively through these divisions and deposits in each a part of its heat. During the process  $da$  the fluid passes again through the divisions but in a reverse direction, from the coldest to the hottest. This ideal process can only be roughly realized as the regenerator has a limited conductivity, permits the flow of heat between adjacent sections, and can only store up heat if the fluid is much hotter and refund it if the fluid is much colder than itself. Therefore only the upper portion of the line  $bc$  can actually be utilized to refund heat gratuitously along the lower portion of  $da$ . Hence in practice it should be possible to obtain a higher efficiency with an engine working on the Carnot cycle, because those working on the isodiabatic cycle have the added losses of the regenerator. But theoretical efficiency is only one of several factors entering into the design of an actual engine and may be more than balanced by the increased size, cost, strength, etc., required to attain it.

A different type of regenerator is described by Swinburne. The engine consists of a primary cylinder to perform the external work and a secondary one to act as a regenerator. Each cylinder possesses sides and



piston absolutely impermeable to heat, but an end which is a perfect heat conductor. The cycle of the working fluid in the primary cylinder consists of two isothermal and two constant-volume processes. Placed in contact with a source of heat of constant temperature  $T_1$  the fluid expands isothermally up to the end of the stroke; then removed from the source of heat, heat is extracted at constant volume until the temperature has fallen to  $T_2$ ; next placed in contact with a refrigerator of constant temperature  $T_2$  the working fluid is compressed isothermally until the initial volume is regained. The second cylinder operates only during the constant volume changes. With its working substance compressed into the clearance space and at the temperature  $T_1$  its conducting end is brought into contact with the corresponding end of the primary cylinder, just as the latter is removed from the source of heat. The two charges in their respective cylinders are thus maintained separate while the temperature is always equalized. The piston in the secondary cylinder now moves forward, performing external work at the expense of the internal energy of its own charge and of the heat received from the primary cylinder,  $c_v(T_1 - T_2)$ . This process is to occur so slowly that only an infinitesimal drop of temperature exists throughout the combined masses. When not in contact with the main cylinder the auxiliary remains inactive. For its return stroke the secondary is placed in contact with

the primary as the latter is removed from the refrigerator, and external work of an amount equal to that developed during the forward stroke must be performed upon it, thus gradually restoring its own temperature and that of the primary by returning to it the heat,  $c_v(T_1 - T_2)$ .

Thus while the entropy of the working charge is decreasing along  $bc$  (Fig. 4) that of the auxiliary charge is increasing an equal amount along  $bc'$ , and while that of the auxiliary charge decreases along  $d'a$  (curve  $c'b$

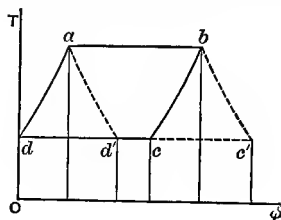


FIG. 4.

moved to the left) the entropy of the working charge undergoes an equal increase at the same temperatures. Thus during the isodiabatic processes the combined entropies of the two charges possess a constant value.

**Decrease in Efficiency due to Irreversibility.**—In all actual engines where the charge enters and leaves the cylinder each cycle, a small difference of pressure and hence of temperature must always exist between the fluid in the supply-pipe and in the cylinder, and between that in the cylinder and in the exhaust-pipe. Thus, let  $T_1$  and  $T_2$  (Fig. 5) represent the temperatures of

source and refrigerator; then, for a Carnot cycle  $abcd$  represents the heat utilized, while  $dcgh$  represents that rejected. If it be assumed that no loss of heat is experienced by the working fluid during admission, but simply the drop in temperature  $T_1 T_1'$ , then area  $a'b'g'h$  must equal area  $abgh$ . Similarly, the heat rejected at  $T_2'$  from the actual engine (area  $d'c'g'h$ ) would at the temperature  $T_2$  be equal to the area  $dee'h$ , and thus exceeds that rejected in the ideal case by the

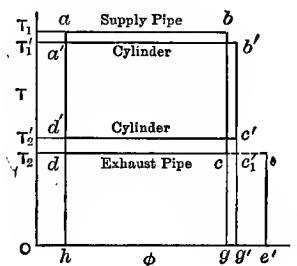


FIG. 5.

area  $cee'g$ . The efficiency of the actual cycle is therefore reduced by these irreversible processes.

It is important to notice that the increased exhaust subdivides into the two areas  $cc_1'g'g$  and  $c_1'ee'g'$ , which represent the losses incurred during admission and exhaust respectively. Thus if there were no throttling during exhaust the adiabatic expansion would be from  $b'$  to  $c_1'$ , and therefore  $cc_1'g'g$  must represent the loss due to throttling at admission alone. The loss is in each case equal to the increase in entropy during the

throttling, multiplied by the lowest available temperature. Thus it follows that any irreversibility, which is always accompanied by a growth of entropy, must cause a decrease in the thermal efficiency of a cycle. Of course, the net efficiency is always still further reduced by actual heat losses due to conduction and radiation.

*Therefore in working between any two temperatures the highest possible efficiency will not be attained unless all the heat received is taken in at the upper temperature, and all the heat rejected is given out at the lower temperature. The only exception to this is in the case of the isodiabatic cycles already considered.*

## CHAPTER II.

### THE TEMPERATURE-ENTROPY DIAGRAM FOR PERFECT GASES.

A PERFECT gas is defined as a substance whose specific volume, pressure, and temperature satisfy the equation

$$pv = RT,$$

and whose specific heat at constant pressure,  $c_p$ , and at constant volume,  $c_v$ , are constants.

Starting from the equation for the conservation of energy,

$$dQ = A(dE + pdv), \quad . \quad . \quad . \quad . \quad (1)$$

for a change of condition at constant volume we obtain

$$dQ = AdE = c_v dT;$$

whence 
$$E = \frac{c_v}{A}T + \text{constant}. \quad . \quad . \quad . \quad . \quad (2)$$

The internal energy of a perfect gas is thus seen to depend solely upon the absolute temperature, so that an isothermal process is also an isodynamic one.

Let us consider next a unit weight of gas in the condition  $(pvT)$ , and let the temperature increase from  $T$  to  $T_1$ , once at constant volume and a second time at constant pressure. In the first case the condition

is changed from  $(pvT)$  to  $(p_1vT_1)$  by the addition of the heat,  $c_v(T_1 - T)$ ; in the second case from  $(pvT)$  to  $(pv_1T_1)$  by the addition of the heat  $c_p(T_1 - T)$ .

Since the change of temperature is the same in both cases the increase in internal energy is the same and therefore the difference between the heat added in the two cases must represent the difference in the external work performed. Then we have

$$\begin{aligned}(c_p - c_v)(T_1 - T) &= Ap(v_1 - v), \\ &= AR(T_1 - T),\end{aligned}$$

$$\text{whence} \qquad c_p - c_v = AR = c_v(k - 1). \quad . \quad . \quad (3)$$

Substituting, this relation in the expression for internal energy gives

$$E = \frac{pv}{k-1} + \text{const.} \quad . \quad . \quad . \quad (4)$$

The entropy for any condition can now be determined without difficulty. Since

$$\begin{aligned}dQ &= c_v dT + A p dv; \\ \phi &= \int \frac{dQ}{T} = c_v \int \frac{dT}{T} + A \int p \frac{dv}{T} + \text{constant}, \\ &= c_v \int \frac{\frac{1}{R}(pdv + vdp)}{\frac{pv}{R}} + AR \int \frac{dv}{v} + \text{constant}, \\ &= c_v [\log_e v + \log_e p] + (c_p - c_v) \log_e v + \text{constant}, \\ &= c_v \left[ \log_e p + \frac{c_p}{c_v} \log_e v \right] + \text{constant}, \\ &= c_v \log_e pv^k + \text{constant}_1 \quad . \quad . \quad . \quad (5a)\end{aligned}$$

By use of the characteristic equation  $pv=RT$ , either  $p$  or  $v$  can be eliminated so that the entropy may also be expressed as

$$\phi = c_v \log_e T v^{k-1} + \text{constant}_2, \quad . \quad . \quad (5b)$$

$$\phi = c_p \log_e T p^{\frac{1-k}{k}} + \text{constant}_3 \quad . \quad . \quad . \quad (5c)$$

THE fundamental heat equations for a perfect gas are

$$\left. \begin{aligned} dQ &= c_v dT + (c_p - c_v) T \frac{dv}{v}, \\ dQ &= c_p dT - (c_p - c_v) T \frac{dp}{p}, \\ dQ &= c_v T \frac{dp}{p} + c_p T \frac{dv}{v}, \end{aligned} \right\} . \quad . \quad . \quad . \quad (6)$$

$$\text{and} \quad c_p - c_v = AR, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which for reversible processes give three different expressions for entropy:

$$\left. \begin{aligned} \phi_2 - \phi_1 &= c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{v_2}{v_1}, \\ \phi_2 - \phi_1 &= c_p \log_e \frac{T_2}{T_1} - (c_p - c_v) \log_e \frac{p_2}{p_1}, \\ \phi_2 - \phi_1 &= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}. \end{aligned} \right\} . \quad . \quad (8)$$

The curves for constant volume and constant pressure, respectively, in the  $T\phi$ -diagram become

$$\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1} \quad . \quad . \quad . \quad . \quad (9)$$

and 
$$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad (10)$$

If  $p=f(v)$  be the  $pv$ -projection of any curve on the characteristic surface  $pv=RT$ ; to find the  $T\phi$ -projection of the same curve it is only necessary to find  $\frac{v_2}{v_1}=f_1(T)$  or  $\frac{p_2}{p_1}=f_2(T)$  from the above equations and to substitute them in the first and second forms, respectively, of equations (8).

The most general form of  $p=f(v)$  with which the engineer is concerned is that of an indicator card, namely,

$$pv^n = p_1 v_1^n = \text{a constant}, \quad . \quad . \quad . \quad . \quad (11)$$

where the exponent is determined from any two points by means of the formula

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1}.$$

The respective projections of equation (11) on the  $Tv$ - and  $Tp$ -planes, as found by combination with the characteristic equation, are

$$T_1 v_1^{n-1} = T_2 v_2^{n-1} = \text{a constant} \quad . \quad . \quad . \quad (12)$$

and 
$$T_1 p_1^{\frac{1-n}{n}} = T_2 p_2^{\frac{1-n}{n}} = \text{a constant}, \quad . \quad . \quad (13)$$

whence it follows that

$$\frac{v_2}{v_1} = \left( \frac{T_1}{T_2} \right)^{\frac{1}{n-1}} \quad \text{and} \quad \frac{p_2}{p_1} = \left( \frac{T_1}{T_2} \right)^{\frac{n}{1-n}}. \quad . \quad . \quad (14)$$



The substitution of equations (14) in the corresponding forms of equations (8) gives

$$\begin{aligned}\phi_2 - \phi_1 &= c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \left( \frac{T_1}{T_2} \right)^{\frac{1}{n-1}} \\ &= \left( c_v - \frac{c_p - c_v}{n-1} \right) \log_e \frac{T_2}{T_1} = c_v \cdot \frac{n-k}{n-1} \cdot \log_e \frac{T_2}{T_1}, \quad (15)\end{aligned}$$

or

$$\begin{aligned}\phi_2 - \phi_1 &= c_p \log_e \frac{T_2}{T_1} - (c_p - c_v) \log_e \left( \frac{T_1}{T_2} \right)^{\frac{n}{1-n}} \\ &= \left( c_p - \frac{n(c_p - c_v)}{n-1} \right) \log_e \frac{T_2}{T_1} = c_v \cdot \frac{n-k}{n-1} \cdot \log_e \frac{T_2}{T_1}. \quad (15)\end{aligned}$$

Equations (11), (12), (13), and (15) represent simply different projections of the same curve on the characteristic surface upon different planes.

From equation (2), p. 4, for perfect gases,

$$d\phi = c \frac{dT}{T}, \quad \text{or} \quad \phi_2 - \phi_1 = c \log_e \frac{T_2}{T_1}. \quad (16)$$

Comparison of equations (15) and (16) shows that the specific heat for the general expansion  $p_1 v_1^n = p_2 v_2^n$  is

$$c = c_v \frac{n-k}{n-1}; \quad . \quad . \quad . \quad . \quad (17)$$

*i.e., the specific heat of any expansion of a perfect gas,  $pv^n = \text{constant}$ , can be expressed as a function of the specific heat at constant volume, the ratio of  $c_p$  and  $c_v$ , and the exponent  $n$  of the expansion.*

Let us consider equations (11), (15), and (17) for the following special cases:

(1) For  $T = \text{const.}$ ,  $pv = RT = \text{constant}$ , and  $n = 1$ .

Equation (17) becomes  $c = c_v \cdot \frac{1-k}{1-1} = \infty$ ; i.e., for an isothermal change the heat capacity of the substance is infinite; that is to say, no matter how much heat is added the temperature will not change.

Equation (15) becomes  $\phi_2 - \phi_1 = \infty \cdot 0$ ; that is, the value of  $\phi$  may undergo any change whatever and the  $T\phi$ -curve simply becomes  $T = \text{const.}$

(2) For  $\phi = \text{constant}$ , equation (15) gives  $n = k$ , and equation (17) becomes  $c = c_v \frac{k-k}{k-1} = 0$ ; i.e., for an isentropic change the heat capacity of the substance is *nil*; that is to say, the temperature of the substance can be increased or diminished without the addition or subtraction of heat *as heat*. Equation (11) becomes  $pv^k = \text{constant}$ .

(3) For  $p = \text{constant}$ , equation (11) becomes  $v_1^n = v_2^n$ , hence  $n$  equals zero.

From (14)  $c = c_v \frac{0-k}{0-1} = c_v \cdot \frac{c_p}{c_v} = c_p$ , and equation (15) becomes  $\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1}$ , as already found in equation (10).

(4) For  $v = \text{constant}$ , the only value of  $n$  which will satisfy  $p_1 v_1^n = p_2 v_1^n$  is  $n = \infty$ ; so that equation (11) becomes  $v = \text{constant}$ .

Equation (17) gives  $c = c_v \frac{\infty - k}{\infty - 1} = c_v$ , whence from equation (15)  $\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1}$ , as previously found in equation (9).

We are now in a position to transfer any curve from the  $pv$ -plane to the  $T\phi$ -plane as soon as we know the value of  $c_v$  for the given gas.

In Fig. 6 let  $abcd$  represent a cycle consisting of two

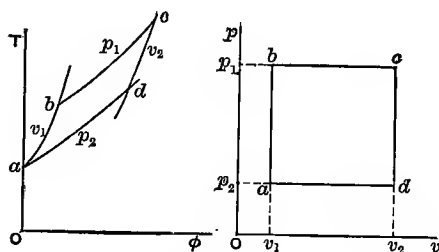


FIG. 6.

curves of constant volume and two of constant pressure indicated by a rectangle in the  $pv$ -plane. In the  $T\phi$ -plane start with the value of the entropy at  $a$  as the zero-point. The curve  $ab$  will be of the nature shown, becoming steeper as  $T$  increases because the subtangent at any point represents the value of  $c_v$  and this is a constant for perfect gases. Arriving at  $b$ , the curve of constant pressure will assume some such position as shown.  $bc$  will not be as steep as  $ab$  at the point of intersection because  $c_p > c_v$ , i.e. the subtangent of  $bc$

is greater than the subtangent of  $ab$ . Two similar curves  $cd$  and  $da$  complete the cycle.

All possible variations of the curves  $pv^n = \text{constant}$ , or  $\phi_2 - \phi_1 = c_v \frac{n-k}{n-1} \log_e \frac{T_2}{T_1}$ , are summarized in the following table and diagrams:

$pv$ -coordinates.		$T\phi$ -coordinates.	
$n$	Form of the Curve.	$c_v \cdot \frac{n-k}{n-1}$	Form of the Curve.
0	$p = \text{constant}$	$c_p$	{ $T$ and $\phi$ increase or decrease together
$0 < n < 1$	$pv^n = \text{constant}$	$> c_p$	
1	$pv = \text{constant}$ (isothermal)	$\infty$	$T = \text{constant}$ *
$1 < n < k$	$pv^n = \text{constant}$	{ Negative	{ $T$ increasing and $\phi$ decr'sing
$k$	$pv^k = \text{constant}$ (isentropic)	0	
$k < n < \infty$	$pv^n = \text{constant}$	$< c_v$	
$\infty$	$v = \text{constant}$	$c_v$	{ $T$ and $\phi$ increase or decrease together

\* During an isothermal change of a perfect gas all the heat added performs external work, hence in the diagrams the isodynamic lines are superimposed upon the isothermals.

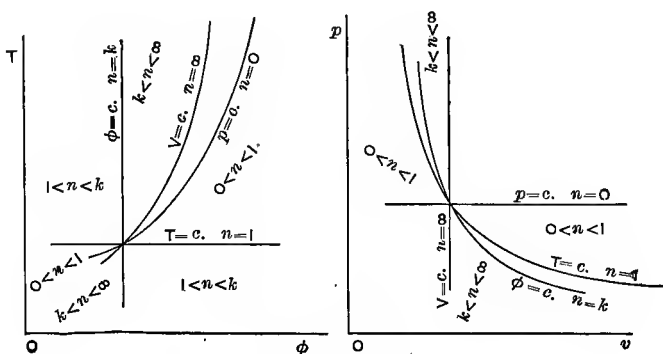


FIG. 7.

**The  $T\phi$ -Projection of any  $pv$ -Curve.**—In case the curve  $p=f(v)$  does not possess an equation of the simple form  $pv^n=c$ , and it thus becomes more difficult to determine the corresponding expression  $\phi=f_1(T)$ , it will sometimes prove simpler to avoid the analytical solution and to find the desired curve graphically by plotting it point by point at the intersections of the proper constant pressure and constant volume curves.

This may be done the more readily since the curves of constant pressure and constant volume

$$\phi = c_p \log_e T + c_1 \quad \text{and} \quad \phi = c_v \log_e T + c_2$$

are of such a character that the intercepts upon isothermals made by any two constant pressure or constant volume curves are of equal lengths and equal respectively to

$$\Delta\phi = AR \log_e \frac{v_2}{v_1} \quad \text{and} \quad \Delta\phi = AR \log_e \frac{p_1}{p_2}.$$

Hence to draw a series of constant pressure or of constant volume curves in the  $T\phi$ -plane the following procedure is necessary and sufficient:

(1) Determine accurately the contour of the two curves

$$\phi = c_p \log_e T \quad \text{and} \quad \phi = c_v \log_e T.$$

(2) Construct a template of both curves.

(3) Determine

$$\Delta\phi = AR \log_e \frac{v_2}{v_1} \quad \text{and} \quad \Delta\phi = AR \log_e \frac{p_1}{p_2}$$

for the necessary pressures and volumes.

(4) Along the isothermal passing through  $p_1v_1$ , lay off the values of  $\Delta\phi$  determined in (3).

(5) Using the templates draw through these points the corresponding pressure or volume curves.

Having these base lines once constructed, any irregular curve,  $p=f(v)$ , may be at once transferred from the  $pv$ - to the  $T\phi$ -plane by noting the values of pressure and volume at a sufficient number of points to give a smooth curve.

**Relative Simplicity of the  $T\phi$ - and  $pv$ -Projections.**—In graphical work where it may be necessary to plot isothermals and reversible adiabatics as well as constant pressure and constant volume curves, it is simpler to use the  $T\phi$ - than the  $pv$ -plane. This is at once evident from the following tabulation:

Curves.	$pv$ -plane.	$T\phi$ -plane.
Constant pressure . . . . .	Horizontal lines. . .	$\phi = c_p \log_e T + c_1$ The curves are all alike
" volume . . . . .	Vertical lines . . . . .	$\phi = c_v \log_e T' + c_2$ The curves are all alike
" temperature . .	$\begin{cases} pv=c \\ \text{No two curves alike} \end{cases}$	Horizontal lines
" entropy . . . . .	$\begin{cases} pv^n=c \\ \text{No two curves alike} \end{cases}$	Vertical lines

. **The Logarithmic Curve  $y=\log x$ .**—The logarithmic curve forms the basis not only of the graphical method

for plotting any polytropic curve in the  $pv$ -plane, but also of the method for projecting any curve from the  $Tv$ - and  $pv$ -planes into the  $T\phi$ -plane. Such a curve can be constructed very quickly from a logarithmic table, and in case much of this work must be done a

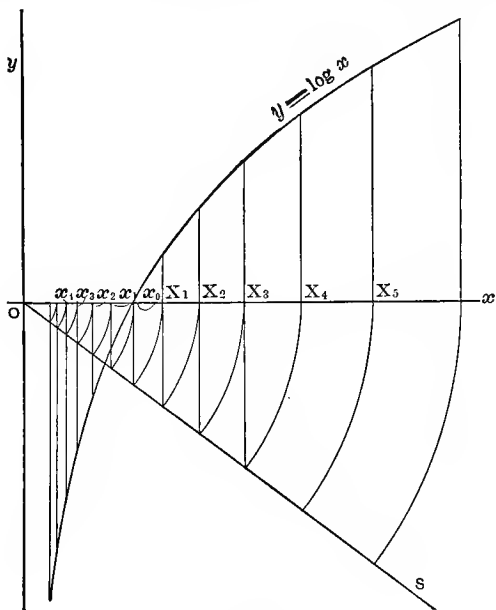


FIG. 8.

template can be constructed and the curve thus be drawn whenever needed.

In case logarithmic tables are not at hand and even if they are, and some arbitrary value of  $x$  is assumed as unity, the following graphical method for constructing the logarithmic curve is often of great convenience.

Let  $Ox_0$  (Fig. 8) represent the arbitrary unit of  $x$ . Through  $O$  draw any straight line  $OS$ . Then starting with  $O$  as a centre and  $OX_0$  as the first radius, construct the series of arcs and perpendiculars, determining the series of values,

$$x_n \dots x_4, x_3, x_2, x_1, x_0, X_1, X_2, X_3, \dots X_n.$$

From the series of similar triangles these quantities are related as follows:

$$\frac{1}{m} = \frac{x_n}{x_{n-1}} = \dots = \frac{x_4}{x_3} = \frac{x_3}{x_2} = \frac{x_2}{x_1} = \frac{x_1}{x_0} = \frac{x_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3} = \dots = \frac{X_{n-1}}{X_n}$$

whence $x_1 = m^{-1}x_0$	$X_1 = mx_0$
$x_2 = m^{-1}x_1 = m^{-2}x_0$	$X_2 = mX_1 = m^2x_0$
$x_3 = m^{-1}x_2 = m^{-3}x_0$	$X_3 = mX_2 = m^3x_0$
$\vdots \quad \vdots \quad \vdots$	$\vdots \quad \vdots \quad \vdots$
$x_n = m^{-1}x_{n-1} = m^{-n}x_0$	$X_n = mX_{n-1} = m^nx_0$

Taking the logarithms and remembering that  $\log x_0 \neq 0$  and setting  $\log m = D$ , we obtain

$$\log x_n = \log m^{-n}x_0 = -n \cdot \log m = -n \cdot D = \log \frac{x_n}{x_0}$$

$$\vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots$$

$$\log x_3 = \log m^{-3}x_0 = -3 \log m = -3 \cdot D = \log \frac{x_3}{x_0}$$

$$\log x_2 = \log m^{-2}x_0 = -2 \log m = -2 \cdot D = \log \frac{x_2}{x_0}$$



$$\log x_1 = \log m^{-1}x_0 = -1 \quad \log m = -1 \quad D = \log \frac{x_1}{x_0}$$

$$\log x \quad \quad \quad \vdots \quad \quad \quad \vdots \quad = \quad O = \log \frac{x_0}{x_0}$$

$$\log X_1 = \log mx_0 = +1 \quad \log m = +1 \quad D = \log \frac{X_1}{x_0}$$

$$\log X_2 = \log m^2x_0 = +2 \quad \log m = +2 \quad D = \log \frac{X_2}{x_0}$$

$$\log X_3 = \log m^3x_0 = +3 \quad \log m = +3 \quad D = \log \frac{X_3}{x_0}$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\log X_n = \log m^nx_0 = +n \quad \log m = +n \quad D = \log \frac{X_n}{x_0}$$

This method of construction has thus given a series of points  $x$  such that the logarithms of any two successive points of the series differ by a common amount  $D$ . Therefore to construct the desired logarithmic curve  $y = \log x$ , starting at  $x_0$ , lay off on the vertical lines through  $X_1, X_2, X_3, \dots X_n$ , and  $x_1, x_2, x_3, \dots x_n$  the distances  $D, 2D, 3D, \dots nD$  and  $-D, -2D, -3D, \dots -nD$ , respectively, and connect them with a smooth curve.

In case any multiple of this curve is desired, such as  $y = a \cdot \log x$ , as, for example,  $\Delta\phi = c_v \log_e \frac{T}{T_0}$ , the ordinates of the curve  $y = \log x$  may each be multiplied by  $a$  and the product  $aD, 2aD, \dots$ , laid off as before. A

simple and more convenient method is that shown in Fig. 9.

Lay off from  $O$  the distance unity ( $x_0=1$ ), from  $x_0$  drop a perpendicular equal to  $a$  units. Draw  $OA$ . Then from similar triangles if  $OB$  equal  $\log x$ ,  $BC$  will equal  $a \cdot \log x$ . Therefore to determine  $a \log x = y$  for any value of  $x$ , take  $\log x$  from the log-curve with

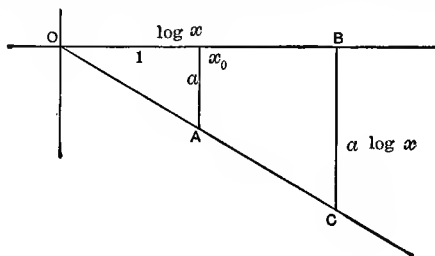


FIG. 9.

the dividers and locate  $B$ . Then measure  $BC$  with the dividers.

**Graphical Construction of the Polytropic Curves ( $pv^n=c$ ) in the  $pv$ -Plane.**—To construct the polytropic curves choose any convenient volume, as  $OA$  (Fig. 10), for unit length, and then through this point draw in any convenient manner the logarithmic curve  $p=\log v$ .

From the origin lay off to the left  $OA'=OA$ , and then  $A'B=-n$ . Draw the straight line  $OB$ , and the constant-volume curve through  $A$ . Choose any number of points on this line, as 1, 2, 3, . . . , according to the number of curves desired, and connect each point by a straight line to the origin.



through these points intersecting  $v_1$  at  $p_1, p_2, p_3, \dots$ . These are the desired points.

The proof of this is simple. From similar triangles,

$$\begin{aligned} v_1' a_1 : -\log v_1 &= -n : -1, \\ v_1' a &= -n \log v_1 = \log v_1^{-n} = bb'. \end{aligned}$$

$$\therefore Ob = v_1^{-n}.$$

Again from similar triangles,

$$\begin{aligned} bh_1 : Ob &= A1 : OA, \\ bh_1 : v_1^{-n} &= p_A : v_A^n; \\ bh_1 &= v_1^{-n} \cdot p_A v_A^n = p_1. \end{aligned}$$

To continue the curves to the left of  $A$ , prolong  $BO$  into  $pv$ -plane, as shown, and lay off  $\log v$  to the right of  $O$ . Then proceed as before.

This same method can be used to plot the  $Tv$ -projection of the polytrope,  $Tv^{n-1} = c$ , provided the auxiliary line  $OB$  is determined by the coordinates  $[-1, -(n-1)]$  instead of  $(-1, -n)$ .

This construction may be used backwards to determine  $n$  for a given polytropic curve, or if the curve is only approximately polytropic a mean value of  $n$  can be determined.

**Construction of the Isothermals,  $pv = c$ , in the  $pv$ -Plane.**—Draw the curves  $p_1 = c$  and  $v_1 = c$ , which bound that portion of the  $pv$ -plane (Fig. 11) in which the isothermals are to be constructed. To construct the isothermal through any point, as  $A$ , draw the curve .

$v=v_a$ . Draw the straight lines  $OI, OII, OIII, \dots$ , which intersect  $v=v_a$  in points 1, 2, 3, 4,  $\dots$ . Draw a series of vertical lines through  $I, II, III, \dots$ , and horizontal lines through 1, 2, 3,  $\dots$ , then the points of intersection  $I1, II2, III3, \dots$ , are points on the desired isothermal.

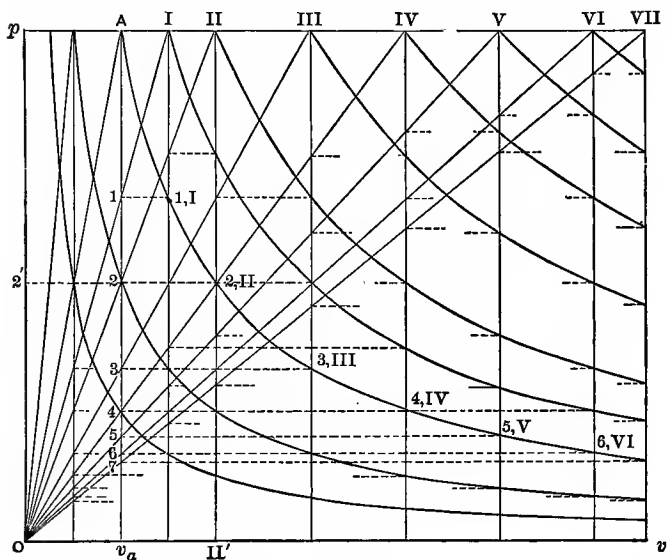


FIG. 11.

The isothermals through the successive points  $I, II, III, \dots$ , can be drawn by using the same radial and vertical lines. The only extra construction necessary being the horizontal lines through the points of intersection of  $I, II, III, \dots$ , with the radial lines.

This construction is very simple if carried out on plotting paper, as then only the radial lines need to be constructed.

The justification for this construction is found in the equation of the curve itself. Thus  $pv=c$  may be construed graphically to mean that the area of the rectangle included between the axes of  $p$  and  $v$  and the constant pressure and constant volume lines through any point on the curve is a constant. Thus take the points ( $A$ ) and ( $2, II$ ) on the isothermal through  $A$ . If these points lie on the same isothermal rectangle  $OA$  must equal rectangle  $O2, II$ . Now  $\triangle OIIp = \triangle OIII'$  and  $\triangle O22' = \triangle O2v_a$  and  $\triangle 2IIA = \triangle 2II(2, II)$ , whence rectangle  $2'A = \text{rectangle } v_a(2, II)$  and therefore rectangle  $OA = \text{rectangle } O(2, II)$ .

**Slide Rule Construction of Isothermals.**—If, in place of drawing isothermals through points already known, it becomes necessary to determine them for definite temperatures the work of plotting may be carried on directly from the slide rule. Thus set the slider to mark the product  $RT$ , and then the volumes corresponding to any desired number of pressures may be read from one setting for each value. The slider must be reset for each new isothermal.

**Representation of  $W$ ,  $Q$ , and  $\Delta E$  in the  $pv$ - and  $T\phi$ -Planes.**—The  $pv$ -plane gives at once the external work performed during, and the  $T\phi$ -plane the heat interchange involved in, any reversible process. To represent the change of internal energy two methods are available in both planes.

(1) Since isothermal and isodynamic lines are coin-

cident it is only necessary to determine  $\Delta E$  per unit increase of temperature and then, by assuming some arbitrary zero of internal energy, to assign values to the different isodynamics. Thus  $\Delta E = \frac{\Delta pv}{k-1} = \frac{R \Delta T}{k-1}$ , and  $\Delta E$  per unit change of temperature equals  $\frac{R}{k-1} = \frac{c_v}{A}$ . Hence the change of intrinsic energy during any process may be found by noting the initial and final state points with reference to the isodynamic lines.

(2) If a substance expands adiabatically, performing work at the expense of its internal energy, and if this process occurs so slowly as to prevent increase of kinetic energy and to permit the maintenance of a uniform temperature, and if there be neither internal nor surface friction, then the expansion will at the same time be isentropic. For such an isentropic expansion the area under the curve in the  $pv$ -plane will not only represent the external work performed but will at the same time be a measure of the decrease of internal energy. Thus between any two points  $a$  and  $b$  upon the same isentrope there must exist the difference of internal energy,

$$E_a - E_b = \int_a^b p dv = \frac{p_a v_a}{n-1} \left[ 1 - \left( \frac{v_a}{v_b} \right)^{n-1} \right],$$

where  $n$  equals the exponent of the adiabatic curve.

If the point  $b$  be removed to infinity there results

$$E_a - E_\infty = \frac{p_a v_a}{n-1};$$

i.e., the decrease in internal energy during frictionless adiabatic expansion from any finite condition  $a$  to infinity is represented by  $\frac{p_a v_a}{n-1}$ . This, however, is not the total value of the internal energy at condition  $a$ , because by definition,

Internal energy = vibration energy plus disgregation energy;

= kinetic energy plus potential energy;

or  $E = S + I,$

and at infinity while the vibration or kinetic energy has become zero with the disappearance of both pressure and temperature, the disgregation or potential energy has assumed its maximum value due to infinite increase in volume. Hence  $E_\infty = S_\infty + I_\infty = 0 + I_\infty = I_\infty$ , and we may write

$$E_a = \frac{p_a v_a}{n-1} + I_\infty.$$

Thus the internal energy of a perfect gas can be determined up to the value of a certain unknown constant,  $I_\infty$ .

In the case of a perfect gas the molecules are supposed to be so far apart as to be beyond the sphere of



mutual attraction, so that the value of  $I_\infty$  is already established at finite distances, and changes in internal energy are directly proportional to changes in temperature.

(a) To find the difference in internal energy between any two state points  $a$  and  $b$  of a perfect gas we have

$$\begin{aligned} E_b - E_a &= \frac{p_b v_b}{k-1} + I_\infty - \frac{p_a v_a}{k-1} - I_\infty = \frac{p_b v_b}{k-1} - \frac{p_a v_a}{k-1} \\ &= \frac{R}{k-1} (T_b - T_a) = \frac{c_v}{A} (T_b - T_a), \end{aligned}$$

or

$$A(E_b - E_a) = c_v T_b - c_v T_a.$$

Interpreted graphically (Fig. 12) this states that the difference between the intrinsic energy of any two

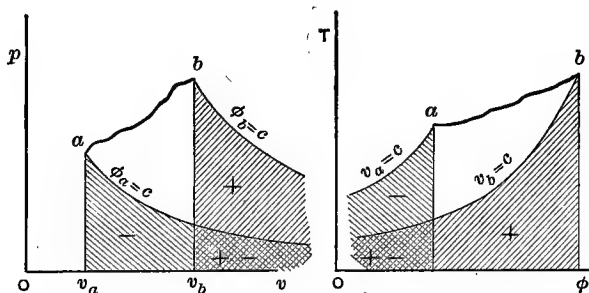


FIG. 12.

state points is represented in the  $pv$ -plane by the difference of the areas under the adiabatics drawn through the respective points and extended to infinity; or the

heat equivalent of the difference in kinetic energy is represented in the  $T\phi$ -plane by the difference of the areas under the constant volume curves drawn through the respective points and extended to infinity.

Here again, although both diagrams are infinite, the  $T\phi$  is the easier to construct accurately.

(b) To avoid the use of the infinite diagram the method shown in Fig. 13 can be used. Suppose it is

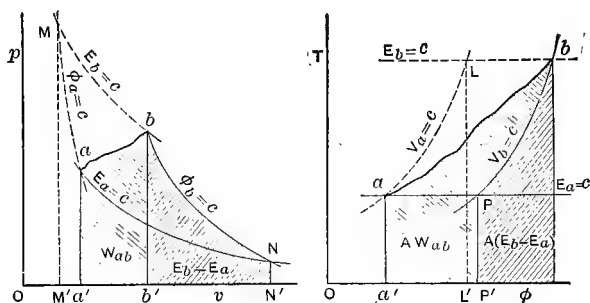


FIG. 13.

desired to find the difference of intrinsic energy between  $a$  and  $b$ . Draw through  $a$  ( $pv$ -plane) the isodynamic  $E_a = c$  and through  $b$  the isentrope  $\phi_b = c$ , intersecting at some point  $N$ . If the gas be imagined to expand isentropically from  $b$  to  $N$  it will develop the work  $bNN'b'$  and suffer a decrease of internal energy  $E_b - E_N$  equal to this work. But by construction  $E_N = E_a$  so that the area under  $bN$  represents the magnitude of the difference of internal energy between  $a$  and  $b$ . An

equivalent solution indicated by dotted lines may also be used.

In the  $T\phi$ -plane a slightly different construction must be used. During a constant volume change no external work is performed and the heat added increases the internal energy. Draw through  $b$  ( $T\phi$ -plane, Fig. 13) the constant volume curve  $v_b=c$ , and through  $a$  the isodynamic  $E_a=c$  intersecting at  $P$ . If the gas be imagined to undergo the change  $Pb$  it will absorb the heat  $c_v(T_b-T_p)=\text{area under } Pb$ , and during this change its internal energy will be increased from  $E_p$  to  $E_b$ , that is, from  $E_a$  to  $E_b$ . Hence the difference between the heat equivalents of the internal energy at the points  $a$  and  $b$  is represented by the area under  $Pb$ . An equivalent solution indicated by dotted lines may also be used.

It follows from the first law of thermodynamics, as applied to reversible processes,  $Q=A \cdot \Delta E + AW$ , that (in the  $pv$ -plane) since the area under  $ab$  represents the work performed and the area under  $bN$  represents the change of internal energy, the algebraic sum of these, or the area under  $abN$ , must represent in work units the heat received. Similarly, in the  $T\phi$ -plane if the area under  $ab$  represents  $Q_{ab}$ , and the area under  $Pb$  represents  $A(E_b-E_a)$ , then the area  $abPP'a'$  must represent  $AW_{ab}$ .

In the case of a perfect gas it is thus possible in both the  $pv$ - and the  $T\phi$ -planes to represent by finite areas

all three terms involved in the statement of the first law of thermodynamics. But again the  $T\phi$ -plane proves itself the simpler of the two, in that one of the two necessary construction curves is a straight line, and the other has only one form for any given gas.

**Graphical Projection of any Curve from the  $pv$ - to the  $Tv$ -Plane.**—From the laws of a perfect gas  $v = \frac{RT}{p}$  or  $v \propto T$  if  $p = \text{constant}$ , it follows that in the  $Tv$ -plane

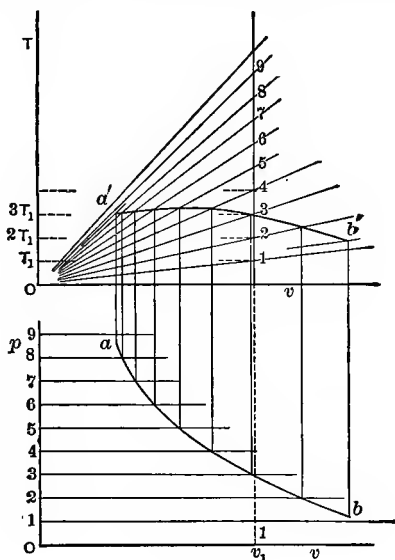


FIG. 14.

a constant pressure curve is represented by a straight line passing through the origin. Let 1, Fig. 14, represent the condition  $T_1$ ,  $v_1$ , and hence the condition  $p_1$ .

Then the line  $O1$  represents the line of constant pressure  $p_1$ . Now if the volume of a perfect gas be maintained constant the pressure is proportional to the temperature, so that the point  $2T_1, v_1$ , represents the pressure  $p_2=2p_1$ . The line  $Op_2$  therefore represents the constant pressure  $2p_1$ . Similarly  $Op_3$  represents  $p=3p_1$ , etc. Thus to obtain the constant pressure curves, lay off on any convenient constant volume curve a series of equal intervals and draw a set of straight lines from these points to the origin. If the scale of  $T$  is already determined the scale of pressure may be determined to correspond, or if these pressures are laid off arbitrarily the temperature scale must be determined to correspond. For ordinary use the latter method is more convenient.

Let a curve  $ab$  be given in the  $pv$ -plane which intersects the pressure curves  $p_1, 2p_1, 3p_1$ , etc., in the points 1, 2, 3, etc., respectively. Each point  $p_1v_1, p_2v_2, p_3v_3, \dots$ , has an unique location in each plane, namely, at the intersection of the corresponding pressure and volume curves. The position in the  $Tv$ -plane may be found by projecting upward along the constant volume curve from  $p$  in one plane to  $p$  in the other.

For any known weight of gas  $T$  is at once determined, but if the quantity is unknown and if the volumes are only known relatively, as in the case of cards taken from hot-air or gas-engines or air-compressors, the scale of  $T$  cannot be determined, but the

projection still gives relative values of the temperature and may thus throw considerable light upon the nature of the operations.

**The Graphical Projection of any Curve from the  $Tv$ -Plane into the  $T\phi$ -Plane.**—The entropy, temperature and volume of a gaseous mixture are connected by the equation

$$\phi = c_v \log_e T + AR \log_e v + \text{constant}.$$

Assuming any condition  $v_0 T_0$  as the reference point the difference of entropy between this reference point and any other point,  $vT$ , is given by

$$\Delta\phi = c_v \log_e \frac{T}{T_0} + AR \log_e \frac{v}{v_0}.$$

Putting the variable factor  $R$  into the left-hand term the expression reduces to

$$\frac{\Delta\phi}{2.303AR} = \frac{1}{k-1} \cdot \log_{10} \frac{T}{T_0} + \log_{10} \frac{v}{v_0}.$$

In the case of diatomic gases when  $k=1.405$  this reduces to

$$\frac{333.5}{R} \cdot \Delta\phi = 2.47 \log_{10} \frac{T}{T_0} + \log_{10} \frac{v}{v_0}.$$

This equation will apply directly to hot-air engine and air-compressor cards, but for gas-engine work the coefficient may need to be modified for each individual case, as  $k$  is a variable, being about 1.38.

It is evident that if we could construct two curves such that the ordinates for any point  $(T, v)$  are respectively equal to  $2.47 \log_{10} \frac{T}{T_0}$  and  $\log_{10} \frac{v}{v_0}$ , then the sum of these two ordinates would equal  $\frac{333.5}{R} \cdot \Delta\phi$ , and by using this value and  $T$  a  $T\phi$ -plot could be constructed. This coefficient could be eliminated from the  $T\phi$ -plot by making  $\frac{333.5}{R} = 1$  the unit of entropy, and then the abscissæ would read directly in units of entropy.

Thus suppose we have the curve  $ab$  (Fig. 15) in the  $Tv$ -plane and desire to find its  $T\phi$ -projection. Choose any point  $T_0v_0$  as the reference point, and then on base lines parallel to the  $T$  and  $v$  axes and located if necessary outside the diagram (the axes themselves may be used if the diagram is not unnecessarily complicated thereby), construct two logarithmic curves (see pp. 23 to 26), such that

$$\Delta y = \log \frac{T}{T_0} \quad \text{and} \quad \Delta y_2 = \log \frac{v}{v_0}.$$

Then by means of the auxiliary construction draw also the curve

$$\Delta y_1 = 2.47 \log \frac{T}{T_0}.$$

Now,

$$\frac{333.5}{R} \Delta\phi = \Delta y_1 + \Delta y_2,$$





and therefore if from any point  $n$  on the curve  $ab$  we obtain by projection upon the two logarithmic curves the distances  $\Delta y_1$  and  $\Delta y_2$ , the sum of these distances laid off to the right of  $\phi_0$  along the isothermal  $T_n$  will locate the point in the  $T\phi$ -plane.

For point  $a$ ,

$$\Delta y_1 = \Delta y_a \quad \text{and} \quad \Delta y_2 = 0,$$

and for point  $b$ ,

$$\Delta y_1 = -\Delta y_b \quad \text{and} \quad \Delta y_2 = +\Delta y_b.$$

After the curve is once constructed the scale of entropy can be chosen by setting  $\frac{333.5}{R} = 1$ .

It is evident when the logarithmic curves are constructed graphically that, as  $D$  is chosen arbitrarily, the scale of the drawing may be anything. In fact, the scale can be suitably chosen by varying the length  $D$ . Furthermore the values  $\Delta y_1$  and  $\Delta y_2$  may be read from the base lines on which the logarithmic curves were constructed or from any convenient parallel line. This would mean increasing or decreasing all the values of  $\Delta\phi$  by the same amount and result simply in a bodily transfer of the  $T\phi$ -projection to the right or left by that amount. This would not affect the value of the diagram, as we are dealing with changes in entropy and not with absolute values. In fact the value of  $\phi$  is infinite so that any point may be taken as the arbitrary zero.

Advantage may thus be taken of these facts to shift the  $T\phi$ -projection parallel to the  $\phi$ -axis so as to obtain the most convenient location on the drawing-paper.

## CHAPTER III.

### THE TEMPERATURE-ENTROPY DIAGRAM FOR SATURATED STEAM.

DUE to the very slight variations in the volume of water with increasing temperature the heat equivalent of the external work is very small, and the difference between the work performed under atmospheric pressure and that which would be performed under a pressure increasing with the temperature according to Regnault's pressure-temperature curve is negligible compared with the heat required to increase the temperature. Hence the value of the specific heat  $c=f(t)$  as determined by Rowland is taken as equivalent to that of the actual specific heat required for the transformation occurring when feed-water is heated in a boiler. Similar statements hold for other fluids.

Therefore the heat required to increase the temperature of a pound of liquid by the amount  $dT$  while the pressure increases by  $dp$  is taken as

$$dq = cdT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence 
$$q_1 = \int_{32^\circ}^{t_1} c dT, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and 
$$\theta_2 - \theta_1 = \int_{t_1}^{t_2} \frac{c dT}{T}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To make the temperature-entropy chart conform to the tables for steam and other saturated vapors, we will plot the increase of entropy and intrinsic energy and the heat added above that of the liquid at freezing-point. Hence for water the zero of the entropy scale will be at  $32^\circ \text{F}$ . Furthermore the chart will be constructed for one pound of water.

Point *a* in Fig. 16 represents the position of a pound of water at  $32^\circ \text{F}$ . on the  $T\phi$ -chart. Starting from here and plotting values of  $\theta$  and  $T$  from the equation

$$\theta = \int_{32^\circ}^t \frac{c dT}{T}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as given in the steam-tables, we obtain the "water-line" *ab*. At any temperature *t* the value of the specific heat *c* is shown by the subtangent *gf*. Further, as we proved in the general case of equation (1), the area under the curve *aejO* represents the heat of the liquid, *q*; that is, the number of heat-units required to warm up the water from  $32^\circ \text{F}$ . to the temperature *t*. This "water-line" is the same kind of a curve as that represented by equation (15) for perfect gases, except that

the specific heat of a gas is a constant, while that of water is a function of the temperature.

If at  $t$  the water has reached the temperature corresponding to the boiler pressure any further increase of heat will cause the water to vaporize under constant pressure and thus there will be an increase of entropy at constant temperature. This will be represented on the chart by the horizontal line  $ed$ , and will continue until all the water is vaporized and a

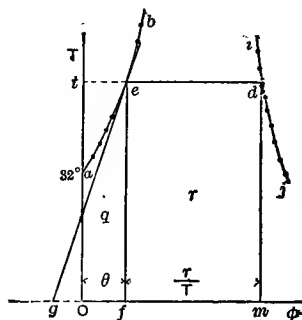


FIG. 16.

condition of dry-saturated steam reached. During this change the increase of entropy will be

$$\phi_2 - \phi_1 = \int_{\phi_1}^{\phi_2} \frac{dQ}{T} = \frac{1}{T} \int_{\phi_1}^{\phi_2} dQ = \frac{r}{T}; \quad . \quad . \quad (5)$$

that is, the length of the line  $ed$  may be found by taking the latent heat of vaporization  $r$  and dividing by the absolute temperature corresponding to  $t$ .

The area under the curve  $ed$  represents the heat added during vaporization or  $r$ .

The area under the curve  $aed$  therefore represents all the heat necessary to be added to a pound of water at  $32^{\circ}$  F. to change it in a boiler into dry steam at the temperature  $t$ , or area  $Oaedm = \lambda = q + r$ .

Similarly the location of the "dry-steam" point may be found for any number of temperatures, thus giving the location of the dry-steam line or saturation curve.

In the area between the water-line and the dry-steam line  $ij$ , or the region of vapor in contact with its liquid, lie all the values given in the steam-tables; to the right of the dry-steam curve lies the region of superheated steam.

Having given the chart with the "water-line" and "dry-steam" line located, and knowing the temperature  $t$  of the steam, it is simply necessary to draw the horizontal line  $ed$  and drop the two perpendiculars  $ef$  and  $dm$  and the tangent  $eg$ ; then the diagram gives at once

$$q, r, \lambda, \theta, \frac{r}{T}, T, \text{ and } c.$$

Let  $e$  in Fig. 17 represent the state point of a pound of water in a boiler under the pressure  $p$  corresponding to the temperature  $t$ . As heat is applied part of the water vaporizes and the state point moves toward  $d$ . At any point as  $M$ , the area under  $eM$  represents the heat already added, while the remaining part, under  $Md$ ,

represents the heat, which must be added to complete the vaporization. That is, for the complete change from water to dry steam the state point travels the distance  $ed$ , and to vaporize one half it would travel one half the distance, etc. Then if  $M$  represents the momentary position of the state point, the ratio  $eM \div ed$  will represent the fractional part of the water already vaporized;

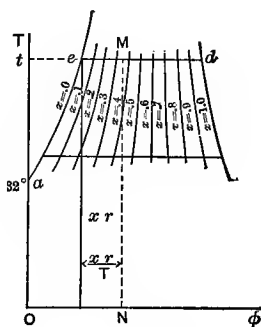


FIG. 17.

that is, the dryness of the mixture,  $x$ , is given by

$$x = \frac{eM}{ed}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence if the line  $ed$  is divided into any convenient number of equal parts (say 10) the value  $x$  can be read directly from the chart as soon as the position of the state point  $M$  is known. In Fig. 17, for example,  $x_M = 0.35$ .

Furthermore, the value of the entropy at  $M$  is represented by that of the liquid at  $e$  and the  $x$ th part of the entropy of vaporization, or

$$\phi_M = \theta_M + \frac{x_M r_M}{T_M};$$

and the total heat at  $M$  equals the heat of the liquid plus the  $x$ th part of the heat of vaporization, or

$$H_M = q_M + x_M r_M,$$

and is represented by the area  $OaeMN$ .

In a similar way the distance  $\frac{r}{T}$  for several temperatures can be divided into the same number of equal parts, and then if all these corresponding points are connected by smooth curves, each curve will represent a change during which the fractional part of the water vaporized is constant. These are known as the  $x$ -curves.

In place of having separate scales of pressure and temperature for the ordinates of the  $T\phi$ -diagram, it is often convenient to take the values of  $p$  and  $t$  from the steam-tables and to plot Regnault's pressure-temperature curve in the second quadrant, as shown in Fig. 18.

Then given any pressure  $p_1$  the corresponding temperature  $t_1$  may be found as indicated or *vice versa*.



Let  $p_1$  be the pressure and  $x_1$  the dryness fraction of a pound of mixture, then on the chart its condi-

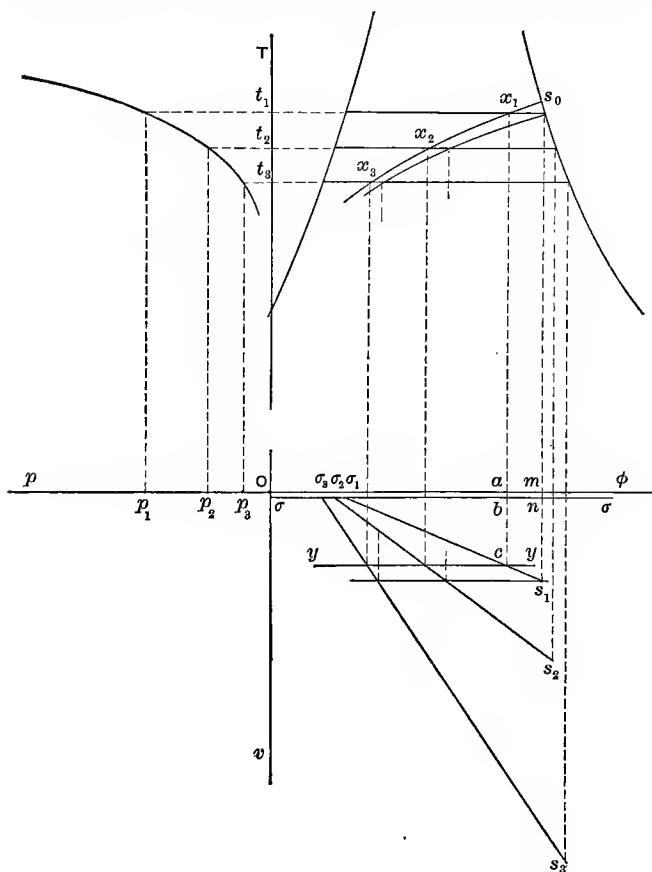


FIG. 18.

tion will be indicated by the point 1. Its volume may be found from the steam-tables by use of the formula  $v = xu + \sigma$ .

To find the location of the state point 3 at any other pressure  $p_3$ , so that  $v_3=v_1$ , proceed as follows:

Draw the axis of volume opposite to that of temperature and lay off along  $Ov$  the distance  $O\sigma$  equal to the volume of one pound of water. The variations of  $\sigma$  with  $t$  may be neglected and  $\sigma$  set equal to 0.016 cubic feet. Draw  $\sigma\sigma$ . Project  $\theta_1$  and  $\theta_1+\frac{r_1}{T_1}$  on the  $\phi$ -axis, and from the latter point lay off the distance  $ms_1$  equal to the volume of one pound of saturated steam as given in the steam-table for  $t_1^\circ$ . Prolong the perpendicular from  $\theta_1$  until it intersects  $\sigma\sigma$  at  $\sigma_1$ . Connect this last point with  $s_1$ . This line  $\sigma_1s_1$  shows the increase of the volume and the entropy during vaporization. Project  $x_1$  upon  $O\phi$  and continue until it intersects the  $v\phi$ -curve at  $c$ . Then  $bc=x_1u_1$  and  $ac=x_1u_1+\sigma_1=v_1$ . Through  $c$  draw  $yy$  parallel to  $O\phi$ . For any pressure  $p_3$  draw the corresponding  $v\phi$ -curve  $\sigma_3s_3$ , and where this intersects  $yy$  the volume will be  $v_3=v_1=x_3u_3+\sigma$ . Projected up this intersects the isothermal  $t_3$  in 3, giving the desired dryness fraction  $x_3$ . Points 1 and 3 have the same volume  $v_1$ . Other points, as 2, etc., may be found and connected with a smooth curve. This will intersect the dry-steam line at some point  $s_0=v_1$ . In this manner similar constant volume curves can be constructed to cover the entire diagram.

Suppose it is required to find the heat necessary

to cause a change from some point  $L$  to an adjacent point  $L+dL$ . (See Fig. 19.)

$$\begin{aligned}\phi &= \theta + \frac{xr}{T} = \int \frac{cdT}{T} + \frac{xr}{T}, \\ d\phi &= \frac{cdT}{T} + \frac{x}{T}dr + \frac{r}{T}dx - \frac{xr}{T^2}dT, \\ dQ &= cdT + xdr + rdx - \frac{xr}{T}dT + (cxdT - cxdT) \\ &= c(1-x)dT + rdx + x\left(c - \frac{r}{T} + \frac{dr}{dT}\right)dT. \quad (7)\end{aligned}$$

To interpret the last term imagine this change to occur along the dry-steam line. Then  $x=1$  and  $dx=0$ , whence

$$dQ = \left(c - \frac{r}{T} + \frac{dr}{dT}\right)dT (= Td\phi). \quad (7a)$$

The comparison of this with eq. (2), p. 4, shows that  $c - \frac{r}{T} + \frac{dr}{dT}$  is the "specific heat" of dry-saturated steam,

$$\text{or} \quad h = c - \frac{r}{T} + \frac{dr}{dT}. \quad (8)$$

The diagram shows at once that  $h$  is negative, i.e. to move along the dry-steam line with increasing temperature heat must be rejected.

Ether has a positive value for  $h$ . This signifies that the saturated-vapor line for ether slants to the right instead of to the left.

Hirn found that steam condensed upon adiabatic expansion and Cazin that it did not condense upon compression. The reason for this is at once clear from Fig. 19. Expanding from  $a$  the reversible adiabatic

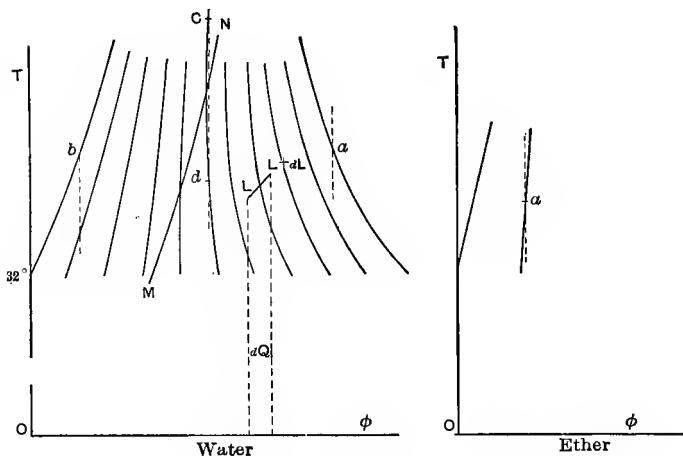


FIG. 19.

line for water cuts successively “ $x$ -lines” of decreasing value, showing condensation. Compressed adiabatically from  $a$  the steam would at once become superheated. Exactly the reverse occurs with ether, condensation occurring during adiabatic compression, superheating during expansion.

If water is permitted to expand adiabatically from  $b$  it is partially vaporized, as shown by the adiabatic line cutting increasing values of  $x$ . Similarly if very wet steam is compressed it condenses.

An inspection of the  $x$ -curves near the value  $x=0.5$  shows that they change from convex to concave and that it is thus possible with water for the reversible adiabatic to cut an  $x$ -curve twice at different temperatures, as at  $c$  and  $d$ ; i.e., it is possible at the end of an isentropic expansion to have the same value of  $x$  as at the beginning. Thus

$$\phi_1 = \phi_2 = \theta_1 + \frac{x_1 r_1}{T_1} = \theta_2 + \frac{x_1 r_2}{T_2}, \quad \text{or} \quad x_1 = \frac{\theta_1 - \theta_2}{\frac{r_2}{T_2} - \frac{r_1}{T_1}}. \quad (9)$$

Consequently there must exist some adiabatic which is tangent to this  $x$ -curve. Above the point of tangency the  $x$ -curve slants to the right and possesses a positive specific heat, below it the curve slants to the left and has a negative specific heat. The values of the specific heat above and below the point of tangency diminish in magnitude as the tangent is approached and at the point of tangency are identical and equal to zero; that is, just there the temperature may be raised without the addition of heat because the change is isentropic.

If these points of zero specific heat are determined for all the  $x$ -curves and connected by the smooth curve  $MN$ , the chart is divided into two portions, such that to the left of  $MN$  the specific heats along the  $x$ -lines are positive, i.e. isentropic expansion will be accompanied by vaporization, and to the right of  $MN$  the specific

heats are negative and isentropic expansion will be accompanied by condensation. The curve  $MN$  is known as the zero-curve.

So far we have located  $p$ ,  $v$ ,  $T$ ,  $\phi$ , and  $x$ , and to make the definition of the point complete it is only necessary to draw a curve of constant intrinsic energy. This could be done by solving  $E_1 = E_2 = E_3 = \text{etc.}$ ,  $= q_1 + x_1 \rho_1 = q_2 + x_2 \rho_2 = q_3 + x_3 \rho_3 = \text{etc.}$ , for the proper values of  $x$  and connecting these by a smooth curve. This would be very laborious, as there does not seem to be any convenient graphical construction. Fortunately it is possible not to draw the isodynamic curves, but to find an area which represents the value of the intrinsic energy for any state point and at the same time to divide  $r$  into two areas proportional to  $\rho$  and  $Apu$  respectively.

The first fundamental heat equation

$$dQ = \left( \frac{dQ}{dt} \right)_v dT + \left( \frac{dQ}{dv} \right)_T dv$$

becomes, when made to conform to the limitations of the first and second laws of thermodynamics,

$$dQ = c_v dT + AT \left( \frac{dp}{dT} \right)_v dv.$$

This is applicable to the case of saturated vapors because the state point is uniquely defined by the intersection of the temperature and volume curves.

Equating the coefficient of the last terms,

$$\left(\frac{dQ}{dv}\right)_T = AT\left(\frac{dp}{dT}\right)_v,$$

a relationship which holds good for any reversible change. During the process of evaporation  $T$  is a constant and

$$\left(\frac{dQ}{dv}\right)_T = \frac{r}{s-\sigma} \quad \text{or} \quad \frac{r}{u} = AT\frac{dp}{dT}, \quad . \quad . \quad (10)$$

whence 
$$Apu = \frac{r}{T} \cdot \frac{p}{\frac{dp}{dT}} = \frac{r}{T} \frac{1}{\frac{1}{p} \frac{dp}{dT}} \quad . \quad . \quad . \quad (10a)$$

Let  $a$  (Fig. 20) be the point of which the intrinsic energy is to be obtained,  $p_a$  is the corresponding pressure. From  $a'$  on Regnault's  $Tp$ -curve draw the tangent  $a'b'$ . From similar triangles it is evident that the subtangent equals  $p \cdot \frac{dT}{dp}$ . Draw  $b'b$  parallel to  $a'a$ .

Then the rectangle  $abcd$  has the area  $p \cdot \frac{dT}{dp} \times \frac{r}{T} = Apu$  (see eq. 10a).

That is,  $abcd$  represents the external heat of vaporization and the rest of  $r$ , namely  $bckf$ , must equal  $\rho$ . Hence the intrinsic energy of the point  $a$  is given by

$$E_a = OgdkO + kcbf = q + \rho.$$









In Fig. 22,  $E_n > E_m$ ; hence this increase of internal energy must have come from the heat added, and subtracting this difference from the total heat added will give the amount remaining for external work. This is positive in the case shown, as was to be expected, as the pressure has fallen and the volume increased.

In case there was a decrease in the internal energy that area would need to be added to the heat area in order to obtain the external work performed.

Having learned to construct and interpret the  $T\phi$ -diagram for saturated vapors we must now resume once more the main object of our investigation, namely, to find the location in the  $T\phi$ -plane of any curve given in the  $pv$ -plane or *vice versa*, so that we may eventually be able to investigate the action of a steam-engine from both its indicator and its temperature-entropy diagrams.

In the case of perfect gases it was possible to use one of the fundamental heat equations and thus obtain a simple analytical expression which could be easily plotted; for saturated steam, however, this process is too cumbersome to be of any service. Fortunately the graphical method offers a solution both simple and elegant.

In constructing the  $T\phi$ -diagram we have already made use of the first, second, and fourth quadrants to express  $T\phi$ ,  $pt$ , and  $v\phi$  variations respectively, and now we have but to take the third or  $pv$ -quadrant and the diagram is complete.

The saturation curve, or curve of constant steam-weight  $ab$  in the  $pv$ -plane, is depicted by the dry-steam curve  $a'b'$  in the  $T\phi$ -plane, Fig. 23. The method of obtaining corresponding points  $aa'$  and  $bb'$  is shown by the projection through the point  $a_1$  and  $b_1$  of the  $pT$ -curve.  $Aa$  and  $Bb$  show the increase in volume of a pound of  $H_2O$  in vaporizing under the constant pressure  $p_a$  and  $p_b$  respectively. This same increase of volume is represented in the  $\phi v$ -plane by the curves  $A''a''$  and  $B''b''$  respectively. Now if only part of the pound is vaporized the actual volume will be indicated by the points, say,  $x_a$  and  $x_b$ . By projection into the  $\phi v$ -plane  $x_a''$  and  $x_b''$  are found. Another projection and we obtain  $x_a'$  and  $x_b'$  as the state points in the  $T\phi$ -plane corresponding to the points  $x_a$  and  $x_b$  in the  $pv$ -plane.

Suppose the pressure and volume of a pound of steam have been determined for some particular part of the stroke by means of the indicator-card and steam measurements. Let  $x_a$  represent such a point. It is now desired to find the corresponding state point in the  $T\phi$ -plane. The procedure is as follows:

Draw through  $x_a$  the curve of constant pressure  $Aa$  and determine by projection its location  $A'a'$  and  $A''a''$  in the  $T\phi$ - and  $\phi v$ -planes respectively. Then project  $x_a$  to  $x_a''$  and finally  $x_a''$  to  $x_a'$  and the desired determination is finished.

As a further problem suppose it is desired to locate

on the  $T\phi$ -plane the curve of constant volume  $x_a x_c$ . The point  $x_a$  is already located at  $x'_a$ . To locate  $x_c$  draw  $Bb$  and find its projections at  $B'b'$  and  $B''b''$ .

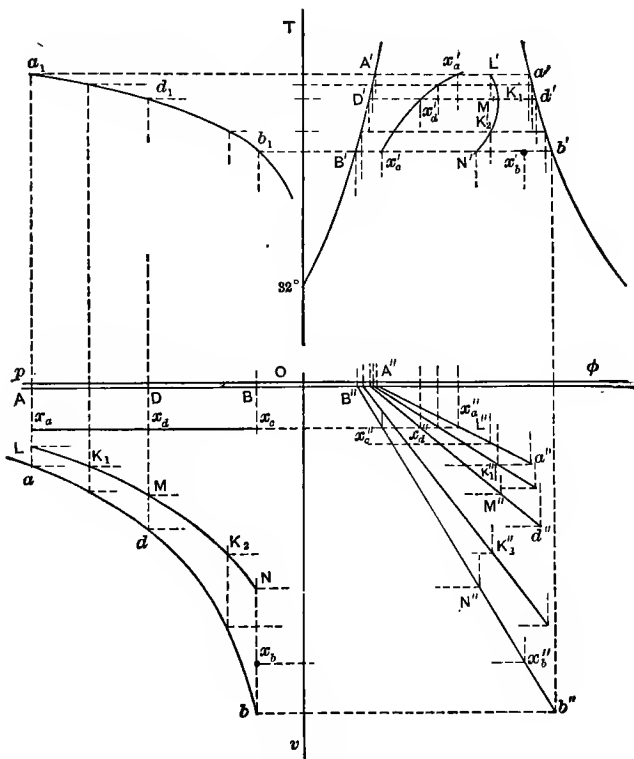


FIG. 23.

Then project  $x_c$  to  $x''_c$  and finally to  $x'_c$ . The two end-points of the curve being determined, any intermediate point as  $x_d$  will be located in the same manner as shown. Thus after a sufficient number of points

are located, the curve of constant volume  $x_a'x_c'$  may be drawn. Naturally if the chart is already provided with constant-volume curves this construction would be unnecessary.

Passing now to the most general problem consider the curve  $LMN$  and suppose its equation in the  $pv$ -plane to be  $pv^n = p_1v_1^n$ . This would correspond to the expansion-line of an indicator-card. It is desired to find the projection of this curve in the  $T\phi$ -plane. The problem resolves simply into the location of a sufficient number of state points, through which a smooth curve is finally to be drawn. To locate  $L$ ,  $M$ , and  $N$  project them on to the corresponding volume curves  $A''a''$ ,  $D''d''$ , and  $B''b''$  of the  $\phi v$ -plane at  $L''$ ,  $M''$ , and  $N''$ , and then finally project to  $L'$ ,  $M'$ , and  $N'$ . To properly determine the curve some intermediate point as  $K$  may be necessary.

**The General Method of Representing  $\Delta E$  and  $W$ .—**The method just explained is simple in application but is confined to saturated vapors alone. It is, however, possible by a line of reasoning similar to that adopted for perfect gases, to develop a method of representing  $\Delta E$  and  $W$  which may be applied to saturated and superheated vapors alike.

Suppose it is desired to find  $E_A$ ,  $xAp_u_A$ , and  $H$  for a pound of saturated vapor whose state point is at  $A$  (Fig. 24). Through  $g$  and  $A$  draw the isodynamic curves  $E_g=0$  and  $E_A=c$ , respectively, and also draw

the curve of constant volume  $V_A = c$  intersecting  $E_g = 0$  in the point  $C$ . During the constant volume change no external work is performed so that the area  $ncAmn$  represents the increase of internal energy in going from  $g$  to  $A$ , and hence is equal to  $q_a + x\rho_A = E_A$ .

It is possible to divide this area into two parts representing  $q_a$  and  $x\rho_A$ , respectively, by finding the intersection  $B$  of the isodynamic  $E_a = q_a$ , drawn through  $a$ ,

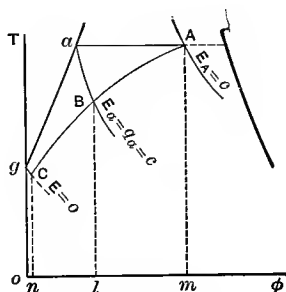


FIG. 24.

with the constant volume curve  $AC$ . Draw the perpendicular  $Bl$ . Then area  $cBln$  represents the heat of the liquid  $q_a$ , and area  $BAmn$  represents the intrinsic energy of vaporization  $x\rho$ .

Since the area  $OgaAmO$  represents the total heat  $H_A$  equal to  $q_a + x\rho_A + xAp_u_A$  it follows that the difference between these two areas, or the area  $OgaACnO$ , must represent the external heat of vaporization.

Fig. 24 illustrated the special problem of representing the quantities of the steam tables by assuming the

substance to start from freezing and to reach the state point  $A$  by travelling along the curve  $gaA$ , and is readily seen to represent a special case of the more general problem illustrated in Fig. 25.

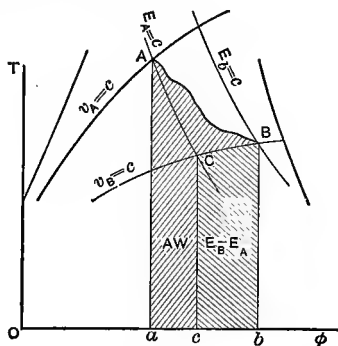


FIG. 25.

Let it be required to find  $Q_{AB}$ ,  $AW_{AB}$ , and  $\Delta E_{AB}$  for any general process such as  $AB$ . Draw through  $A$  the isodynamic curve  $AC$  and through  $B$  the constant volume curve  $BC$  intersecting at  $C$ . Then if the vapor be assumed to undergo the process  $CB$ , the increase in intrinsic energy  $E_B - E_A$  will be represented by area  $CBbc$ . And as  $Q_{AB}$  is represented by area  $ABba$ , the heat equivalent of the external work must be represented by area  $ABCca$ .

It is true that the isodynamic through  $g$  cannot be drawn without knowledge of the properties of water vapor in contact with ice, but this does not affect the validity of this method as applied to the general case just discussed.



## CHAPTER IV.

### THE TEMPERATURE ENTROPY DIAGRAM FOR SUPER-HEATED VAPORS.

THE analytical treatment of superheated vapors, such as steam, ammonia, sulphurous anhydride, etc., is more complicated than that of perfect gases for two reasons:

(1) The characteristic equations are more complicated and but imperfectly known; and

(2) The specific heat at constant pressure and constant volume are no longer constant but follow complicated laws, unknown in most cases and but imperfectly known in others.

At the present time two different equations are being used for superheated steam. Thus Stodola in his *Steam Turbines* makes use of the Battelli-Tumlriz equation  $p(v + 0.135) = 85.1T$ , while Peabody has based his new Entropy Tables upon the equation of Knoblauch, Linde, and Klebe,

$$pv = 85.85T - p(1 + 0.00000976p) \left( \frac{150,300,000}{T^3} - 0.0833 \right).$$

A simplified form of the latter, which does not differ from this by more than 0.8 per cent., has the same

form as the Battelli-Tumlirz, but different values for the constants, viz.,

$$p(v+0.256)=85.85T.$$

Of the many determinations of  $c_p$  the results of only two sets of investigators need be noticed. Those obtained by Thomas and Short, because their values are used in Peabody's Entropy Tables,\* and those of Knoblauch and Jakob, as their values vary most closely in accordance with theoretical predictions:

THOMAS AND SHORT. (Mean value of  $c_p$ .)

Degree of Superheat Fahr.	Pressure Pounds per Square Inch. (Absolute.)						
	6	15	30	50	100	200	400
20°	0.536	0.547	0.558	0.571	0.593	0.621	0.649
50°	0.522	0.532	0.542	0.555	0.575	0.600	0.621
100°	0.503	0.512	0.524	0.537	0.557	0.581	0.599
150°	0.486	0.496	0.508	0.522	0.544	0.567	0.585
200°	0.471	0.480	0.494	0.509	0.533	0.556	0.574
250°	0.456	0.466	0.481	0.496	0.522	0.546	0.564
300°	0.442	0.453	0.468	0.484	0.511	0.537	0.554

KNOBLAUCH AND JAKOB. (Mean value of  $c_p$ .)

Kg. per sq. cm....	1	2	4	6	8	10	12	14	16	18	20
Lbs. per sq. in....	14.2	28.4	56.9	85.3	113.8	142.2	170.6	199.1	227.5	156.0	284.4
Cor. temp. Cent....	99°	120°	143°	158°	169°	179°	187°	194°	200°	206°	211°
Cor. temp Fahr. . .	210°	248°	289°	316°	336°	350°	368°	381°	392°	403°	412°
F.   C.											
212°   100°	0.463										
302°   150°	0.462	0.478	0.515								
392°   200°	0.462	0.475	0.502	0.530	0.560	0.597	0.635	0.677			
482°   250°	0.463	0.474	0.495	0.514	0.532	0.552	0.570	0.588	0.609	0.635	0.664
572°   300°	0.464	0.475	0.492	0.505	0.517	0.530	0.541	0.550	0.561	0.572	0.585
662°   350°	0.468	0.477	0.492	0.503	0.512	0.522	0.529	0.536	0.543	0.550	0.557
752°   400°	0.473	0.481	0.494	0.504	0.512	0.520	0.526	0.531	0.537	0.542	0.547

\* In the 1909 and later editions of the Entropy Tables Peabody uses the values of  $c_p$  obtained by Knoblauch and Jacob.

To obtain the heat required to superheat at constant pressure we must either know how  $c_p$  varies with the temperature at any given pressure and then integrate the expression  $Q = \int_1^2 c_p dT$ , or else we may do what is accurate enough for all engineering work—take a mean value of  $c_p$  and assume this value to be maintained throughout the given temperature range.

Making the same assumption with reference to  $c_p$  the increase in entropy during superheat is given by

$$\Delta\phi_{12} = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{c_p dT}{T} = c_p \log_e \frac{T_2}{T_1}.$$

In superheated as in saturated steam increase in entropy and internal energy as well as total heat added along a constant pressure curve, are all measured from water at 32° F. as an arbitrary zero. Thus if a pound of water at 32° F. is confined under a pressure of  $p$  pounds per square foot, and is then heated to a temperature  $T$  corresponding to this pressure, next vaporized at constant pressure, and finally superheated at constant pressure to some final temperature  $T_s$ , we may express the changes in its entropy, and internal energy and the total heat added by the formulæ,

$$\Delta\phi = \int_{32}^T \frac{cdT}{T} + \frac{xr}{T} + c_p \log_e \frac{T_s}{T}$$

$$Q = H = q + xr + c_p(T_s - T)$$

$$\Delta E = q + xp + c_p(T_s - T) - Ap(v - s).$$

The first term of each expression refers to the warming of the water, the second to its vaporization, and the rest to the process of superheating. Of course, when the steam is superheated  $x=1$ .

There is no method of measuring directly the increase in internal energy during vaporization or during superheating at constant pressure, so that in both cases we are forced to fall back upon the first law of thermodynamics  $Q=A(\Delta E+W)$ , and measure  $Q$  and  $AW$ , thus obtaining  $A\Delta E$  indirectly. Thus  $\rho=r-Apu$ , and similarly the increase of internal energy from a condition of dry steam to any degree of superheat equals  $Q-AW=c_p(T_s-T)-Ap(v-s)$ .

The above equations combined with those for determining the specific volume,

$$v=.016+x(s-.016) \text{ for saturated steam}$$

$$\text{and } pv=85.85T-0.256p \text{ for superheated steam,}$$

make it possible to solve all heat and work problems involving changes of condition in superheated steam, or between saturated and superheated steam.

**Construction of the Constant Pressure Curves in the  $T\phi$ -Plane.**—Starting at the dry-steam line (Fig. 26) at any pressure  $p$  corresponding to the temperature  $T$ , look up in the  $c_p$ -table the mean value of  $c_p$  for this pressure, and, say, 50 degrees superheat. Compute the value of

$$\Delta\phi_1=c_{p_1}\log_e\frac{T+50}{T}.$$

Take from the table the mean value of  $c_p$  for each successive 50 degrees and compute

$$\Delta\phi_2 = c_{p_2} \log_e \frac{T+100}{T+50}, \quad \Delta\phi_3 = c_{p_3} \log_e \frac{T+150}{T+100}, \dots$$

Continue **this** operation until a sufficient number of points has been determined to locate the curve with

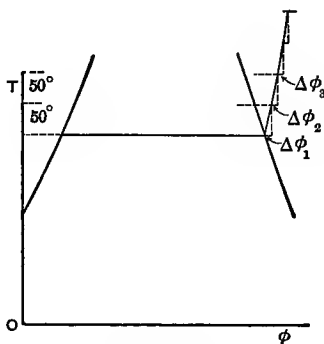


FIG. 26.

the desired accuracy. This operation must be repeated until a sufficient number of lines has been accurately determined to permit of interpolation for the remainder.

If a copy of the entropy tables is at hand it may be used to plot those curves or portions of curves which fall within its limits.

In general these curves are about twice as steep as the water-line, as the specific heat of water is about twice that of superheated steam.

**Construction of the Constant Volume Curves.**—It is possible from the fundamental relation

$$c_p - c_v = \frac{AT}{\left(\frac{dT}{dp}\right)_v \left(\frac{dT}{dv}\right)_p},$$

together with the characteristic equation for superheated steam and the tables of  $c_p$ , to derive (1) an expression for the momentary value of  $c_v$  in terms of the momentary values of  $p$ ,  $v$ ,  $T$ , and  $c_p$ ; or (2) an expression for the mean value of  $c_v$  at any volume for a given increase in temperature. By such a method a table could be constructed for  $c_v$ .

Until a table of values of  $c_v$  has been computed the simplest method of constructing the constant volume curves is as follows:

Substitute the value of the desired volume in the characteristic equation and then solve the equation to obtain the pressures corresponding to a series of temperatures  $T_s$ ,  $T_s + 50$ ,  $T_s + 100$ , . . . From the pressures thus obtained look up the corresponding temperatures of saturated steam and the mean values of  $c_p$  between the dry-steam line and  $T_s$ ,  $T_s + 50$ , . . . From these data compute,

$$J\phi_1 = c_{p_1} \log_e \frac{T_s}{T_1}, \quad J\phi_2 = c_{p_2} \log_e \frac{T_s + 50}{T_2},$$

$$J\phi_3 = c_{p_3} \log_e \frac{T_s + 100}{T_3}.$$

Plot the temperature against the corresponding increase in entropy along the constant pressure curve from the dry-steam line up to the given point for each temperature (Fig. 27). The constant volume curve thus

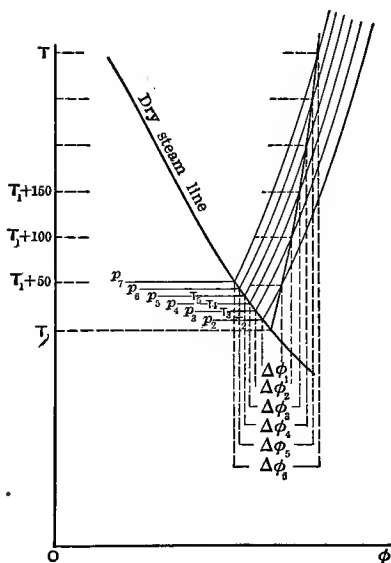


FIG. 27.

determined will of course be steeper than the constant pressure curves because  $c_v < c_p$  and therefore

$$c_v \log_e \frac{T_2}{T_1} < c_p \log_e \frac{T_2}{T_1}.$$

The work involved can be much reduced by using the entropy tables. Look under each value of entropy

for the desired volume and read at once the degrees superheat. Beyond the limits of the tables the above method must be used.

**Isodynamic Curves.**—The isodynamic line for a perfect gas coincides with the isothermal; for a mixture of a liquid and its vapor the divergence is very great, the temperature along the isodynamic dropping rapidly as the entropy increases; for superheated vapors which occupy an intermediate position the isodynamic approaches more and more closely to the isothermal the further the state point is from the dry-vapor line.

To plot the isodynamic  $E=c$  (Fig. 28) extending

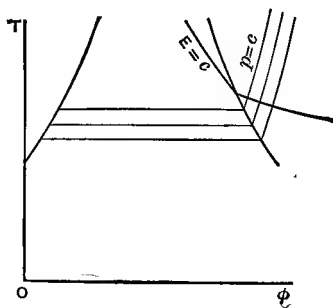


FIG. 28.

from the saturated into the superheated region we have

$$E = q_1 + x_1 \rho_1 = q_2 + \rho_2 + c_p(T_s - T_2) - A p_2(v - s_2).$$

In the saturated region the quality

$$x = \frac{E - q}{\rho}$$



may be obtained for a sufficient number of temperatures by looking up the corresponding  $q$  and  $\rho$ .

In the superheated region it is necessary to find the points of intersection of the isodynamic with several constant pressure curves. The process is cumbersome, as the temperature can only be found by approximation. Thus given  $E=c$  and  $p=c$ , we have

$$E = q + \rho + c_p(T_s - T) - Ap(v - s),$$

where everything is known except  $T_s$ ,  $v$ , and  $c_p$ . But from the characteristic equation

$$v = \frac{85.85T_s}{p} - 0.256.$$

By combination we have

$$E = q + \rho + c_p(T_s - T) - Ap\left(\frac{85.85T}{p} - 0.256 - s\right);$$

whence

$$\left(c_p - \frac{85.85}{778}\right)T_s - c_pT = E - q - \rho - \frac{(s + 0.256)p}{778} \\ = N.$$

Assume  $T_s$  and take the corresponding mean value of  $c_p$  from the table. If the left hand gives a value differing from  $N$  try other values of  $T_s$ .

Having found the temperatures at the points of intersection the points can be at once located, provided the pressure curves are already drawn, otherwise the corresponding entropies must also be computed.

Another method, Fig. 29, involving less computation, is to find the values of  $E$  at several points along a series of isothermals and then by interpolation to connect corresponding values with smooth curves.

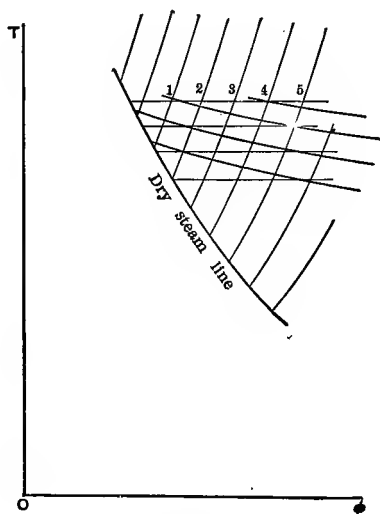


FIG. 29.

Fortunately most if not all problems occurring in engineering practice can be solved without the aid of such curves, so that they are not found on the ordinary diagrams.

It is thus possible to construct five sets of curves, one set each for constant pressure, volume, temperature, entropy, and internal energy changes. As the intersection of any two of these gives a unique location



heated region. Draw through  $A$  the constant pressure curve  $Aba$  and the constant volume curve  $Ac d$ . Draw through  $b$  the constant volume curve  $vb$ . Draw through points  $A$ ,  $b$ ,  $a$ , and  $g$  the isodynamic curves  $E_A$ ,  $E_b$ ,  $E_a$ , and  $E_g$ , cutting the constant volume curve  $v_A$  in the points  $A$ ,  $e$ ,  $h$ , and  $d$ , respectively. Then we have

$H$  represented by area under  $g a b A$ ,

$E_A$  " " " " "  $d h c A$ , or by the  
sum of the areas under  $ga$  and under  $h c A$ ;

$W_{gA}$  represented by area  $O g a b A c d n o$ , or by the  
area  $a' a b A c h i a'$ .

In Fig. 31 let  $AB$  represent any process whatever

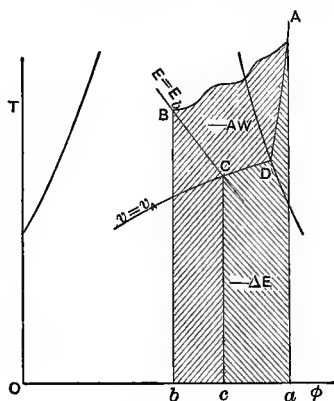


FIG. 31.

which in order to be perfectly general is assumed to start with superheated and to end with saturated vapor. Draw through  $A$  the constant volume curve

$v$  and through  $B$  the isodynamic  $E_B$ , intersecting  $v_A$  at  $C$ . Then the heat rejected during the change  $AB$  is represented by the area  $ABba$ , the decrease in internal energy,  $E_A - E_B$ , by area  $ADCca$ , and the heat equivalent of the work performed upon the substance by area  $ABbcCDA$ .

To obtain the external work during any change measure the area under the curve which represents the change with a planimeter. Then note by reference to the isodynamic lines the initial and final values of the internal energy. Then  $AW = Q - \Delta E$ .

## CHAPTER V.

### THE TEMPERATURE-ENTROPY DIAGRAM FOR THE FLOW OF FLUIDS.

IN dealing with fluids in motion when the velocity of any given mass varies from moment to moment it becomes necessary to introduce a term for the kinetic energy of translation into the mathematical expression for the conservation of energy.

Suppose an expansible fluid to be flowing continually through a conduit of varying cross-section (Fig. 32).

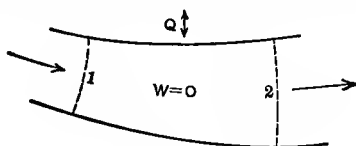


FIG. 32.

If no external work is performed by the fluid other than to push itself forward we know that all the energy carried out across section 2 must be equal to that brought in across section 1 plus the heat received by the fluid in its passage from 1 to 2. If further we assume the conduit to be constructed of a perfect non-

conductor of heat then the process is adiabatic and there must exist the following energy balance per pound,

$$p_1 v_1 + E_1 + \frac{V_1^2}{2g} = p_2 v_2 + E_2 + \frac{V_2^2}{2g}.$$

The increase in kinetic energy between the two sections is therefore

$$\frac{V_2^2 - V_1^2}{2g} = p_1 v_1 - p_2 v_2 + E_1 - E_2.$$

If the process is assumed to be frictionless and therefore in the thermodynamic sense reversible (see p. xi of Introduction) the expansion occurs at constant entropy. That is, the specific volume, temperature and internal energy are found for any given pressure from the equation for frictionless adiabatic expansion,

$$pv^n = \text{constant},$$

where  $n = k = 1.405$  for air and other diatomic gases;

$$n = 1.035 + 0.100x \quad \text{for wet steam} \quad (x = \text{initial quality});$$

$$n = 1.3 \text{ to } 1.33 \text{ for superheated steam.}$$

If then the pressure and volume at section 1 are represented by point 1 in Fig. 33, it follows that the specific volume for any lower pressure may be found upon the constant entropy curve through 1. Thus for pressure  $p_2$  the volume is  $v_2$ . This diagram thus enables

us to give a graphical interpretation to the expression for the increase in kinetic energy between 1 and 2. The first term  $p_1 v_1$  is represented by the rectangle 01, the second term by the rectangle 02. The last two terms together measure the decrease in internal energy between sections 1 and 2 and are represented by the

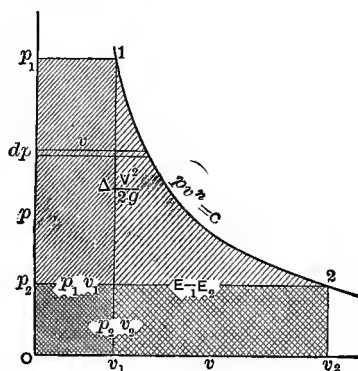


FIG. 33.

work performed during this expansion, even if expended in producing self-acceleration. Therefore  $E_1 - E_2 = \int_1^2 p dv$  is represented by the area under the curve 12. The algebraic sum of these different areas shows that the increase in kinetic energy during frictionless, adiabatic flow is represented by the area between the expansion line and the pressure axis, or

$$\begin{aligned} 4 \frac{V^2}{2g} &= \int_2^1 v dp = \frac{n}{n-1} [p_1 v_1 - p_2 v_2] \\ &= \frac{n}{n-1} \cdot p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]. \end{aligned}$$



This discussion, which up to this point has been perfectly general, must now be carried on independently for each substance.

**Perfect Gases.**—If it is possible to project each point of the  $pv$ -plane uniquely into the  $T\phi$ -plane it follows that a given area in the one plane can be transferred to the other by simply determining its boundary curves

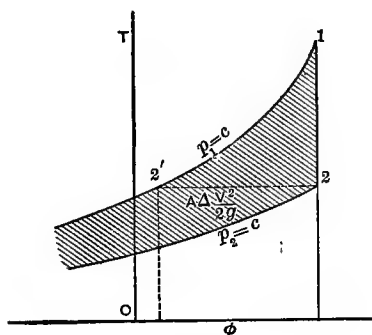


FIG. 34.

in the two planes. The area in the  $T\phi$ -plane will then be the heat equivalent of the corresponding area in the  $pv$ -plane.

Let 12, Fig. 34, be the  $T\phi$ -projection of curve 12 in Fig. 33. The constant-pressure curves through 1 and 2 will intersect at infinity with the curve of zero volume so that the cross-hatched area will represent the heat equivalent of the increase of kinetic energy.

A slight transformation of the above formula makes

it possible to replace the infinite diagram by an equivalent finite one (Fig. 35). Thus from

$$A \frac{V^2}{2g} = \frac{k}{k-1} \cdot (p_1 v_1 - p_2 v_2)$$

it follows that

$$A \cdot A \frac{V^2}{2g} = c_p (T_1 - T_2).$$

$A \cdot A \frac{V^2}{2g}$  is thus represented by the area under the constant pressure curve between the upper and lower

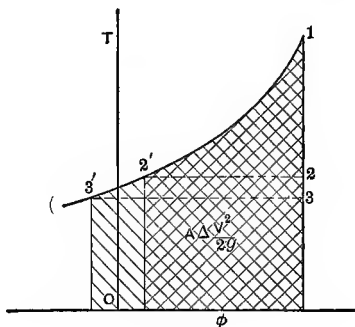


FIG. 35.

temperature levels. Thus in expanding from 1 to 2 the heat equivalent of the increase in kinetic energy is shown by the area under 12'. If the expansion is continued to 3 the further increase in kinetic energy is shown under 2'3', while the total amount is shown under 13'. The same result might have been obtained from Fig. 34 by remembering that all the constant-

pressure curves for a perfect gas have exactly the same form, so that the curve  $2'\infty$  is an exact reproduction of  $2\infty$ , and therefore the total area under 1 minus that under 2 leaves the area under  $12'$ .

An interesting comparison between the flow of a perfect gas and the velocity of a freely falling body can be made from this method of presentation. Imagine the expansion to continue until the final pressure becomes zero, when the volume will be infinite. The kinetic energy will then equal  $c_p T_1$ . This represents the total energy existing in space at 1 due to the presence of the pound of gas, i.e., suppose one pound of air at zero temperature could have been inserted into this space at 1 under pressure  $p_1$  and then heated to  $T_1$ , the total energy thus introduced would be  $c_p \cdot T_1$ . If then we represent this total heat by  $H$ , we notice that the increase in kinetic energy is equal to the decrease in total heat or

$$\frac{V^2}{2g} = 778 \cdot \Delta H, \quad \text{whence} \quad V_2 = \sqrt{2g \cdot 778 \cdot \Delta H} \quad \text{if} \quad V_1 = 0.$$

It is thus seen that the "total heat" head (Erzeugungswärme) plays the same role in the acceleration of fluid flow that gravitational head does in the case of a freely falling body.

**Saturated and Superheated Vapors.**—Before projecting into the  $T\phi$ -plane draw in the liquid ( $\sigma$ ) and dry vapor ( $s$ ) lines (Fig. 36). It is then seen that the



expansion, and the two pressure curves, may be at once located in the  $T\phi$ -plane as indicated in Fig. 37. The little rectangle to the left of the liquid line is retained in the  $pv$ -plane. Area  $abcd$  in the  $T\phi$ -plane is seen to be equal to  $H_1 - H_2$ , and area  $klmn$ , expressed in heat units is  $\frac{(p_1 - p_2)\sigma}{778}$ . Therefore

$$A\Delta\frac{V^2}{2g} = H_1 - H_2 + \frac{(p_1 - p_2)\sigma}{778}.$$

Now  $H + \frac{p\sigma}{778} + E_{32}$  is the total energy (Erzeugungswärme) existing in any space due to the presence of the pound of substance under these conditions. The difference in this total space energy differs by the amount  $\frac{(p_1 - p_2)\sigma}{778}$  from the difference of total heats ( $H_1 - H_2$ ).

Thus if  $V_1 = 0$ ,

$$\begin{aligned} V_2 &= \sqrt{2g[778(H_1 - H_2) + (p_1 - p_2)\sigma]} \\ &= \sqrt{2g \cdot \Delta[778H + p\sigma]} = \sqrt{2g \cdot 778 \cdot \Delta H}, \text{ approx.} \end{aligned}$$

Here again the "total energy" plays the same role as it does with the perfect gases, and thus similar to that played by gravitational head in the case of a freely falling body.

We can now at once write out the formula required for each of the three special cases.

$$\begin{aligned}
(1) \quad A \cdot A \frac{V^2}{2g} &= q_1 + x_1 r_1 - q_2 - x_2 r_2 + A(p_1 - p_2)\sigma; \\
(2) \quad &= q_1 + r_1 + c_p(T_{s_1} - T_1) - q_2 - r_2 \\
&\quad - c_p(T_{s_2} - T_2) + A(p_1 - p_2)\sigma; \\
(3) \quad &= q_1 + r_1 + c_p(T_{s_1} - T_1) - q_2 - x_2 r_2 + A(p_1 - p_2)\sigma.
\end{aligned}$$

For the sake of those who may prefer analytical methods to graphical ones the ordinary manner of deriving these formulæ will be given next.

Starting from the fundamental equation,

$$A \frac{V^2}{2g} = p_1 v_1 - p_2 v_2 + E_1 - E_2,$$

we must substitute in each case the special values for volume and internal energy and then collect terms

$$\begin{aligned}
(1) \quad A \cdot A \frac{V^2}{2g} &= A p_1 (x_1 u_1 + \sigma) - A p_2 (x_2 u_2 + \sigma) + q_1 + x_1 \rho_1 - q_2 \\
&\quad - x_2 \rho_2 \\
&= q_1 + x_1 \rho_1 + x_1 A p_1 u_1 - q_2 - x_2 \rho_2 - x_2 A p_2 u_2 \\
&\quad + A(p_1 - p_2)\sigma \\
&= H_1 - H_2 + A(p_1 - p_2)\sigma; \\
(2) \quad A \cdot A \frac{V^2}{2g} &= A p_1 (v_1 - s_1 + u_1 + \sigma) - A p_2 (v_2 - s_2 + u_2 + \sigma) \\
&\quad + q_1 + \rho_1 - q_2 - \rho_2 + c_p(T_{s_1} - T_1) \\
&\quad - A p_1 (v_1 - s_1) - c_p(T_{s_2} - T_2) + A p_2 (v_2 - s_2) \\
&= q_1 + \rho_1 + A p_1 u_1 + c_p(T_{s_1} - T_1) - q_2 - \rho_2 \\
&\quad - A p_2 u_2 - c_p(T_{s_2} - T_2) + A(p_1 - p_2)\sigma. \\
&= H_1 - H_2 + A(p_1 - p_2)\sigma.
\end{aligned}$$

$$\begin{aligned}
 (3) \quad A \frac{V^2}{2g} &= A p_1(v_1 - s_1 + u_1 + \sigma) - A p_2(x_2 u_2 + \sigma) + q_1 + \rho_1 \\
 &\quad - q_2 - x_2 \rho_2 + c_p(T_{s_1} - T_1) - A p_1(v_1 - s_1) \\
 &= q_1 + \rho_1 + A p_1 u_1 + c_p(T_{s_1} - T_1) - q_2 - x_2 \rho_2 \\
 &\quad + A(p_1 - p_2)\sigma \\
 &= H_1 - H_2 + A(p_1 - p_2)\sigma.
 \end{aligned}$$

If we consider the flow through such conduit or nozzle to be part of a continuous cycle of operations, it becomes necessary to return the water to the boiler, that is, to pump it back again from  $p_2$  to  $p_1$ , the work required being  $A(p_1 - p_2)\sigma$ . Hence if this term be subtracted the difference will represent that part of the increase in kinetic energy which is available for external work, such as driving the rotor of a turbine. Thus,

$$\text{net } A \frac{V^2}{2g} = 778(H_1 - H_2).$$

**Design of a Turbine Nozzle.**—Suppose now we wish to design a nozzle to permit the flow of  $G$  pounds of steam per second. At any cross-section of area  $F$  the necessary and sufficient condition for continuity of flow is that

$$G = F \frac{V}{v} \quad \text{or} \quad \frac{F}{G} = \frac{v}{V}.$$

For any pressure  $p_x$  there can be only one definite value for velocity and specific volume, viz.,  $V_x$  and  $v_x$ , and





Draw  $Od''$  and prolong if necessary until it intersects  $yy$  in  $k$ . From the similar triangles  $Oyk$  and  $Od''n$  it follows that

$$yk : yO = On : nd''$$

or 
$$yk = yO \cdot \frac{On}{nd''} = G \cdot \frac{v_a}{V_a} = F.$$

Conversely to find the pressure which would be obtained at any cross-section, lay off  $yk = F$ , draw  $kO$ , and prolong until it intersects  $MN$  in  $d''$ , from  $d''$  drop the perpendicular  $d''n$  and project  $d'$  back to  $d$ .

The smallest cross-section, or the throat of the nozzle, will be reached when  $Ok$  is tangent to  $MN$ , viz.,  $Ok'$ , giving the value  $F_{throat} = yk'$ .

As  $Ok$  cuts the  $vV$ -curve between  $K$  and  $N$  the value of  $F$  increases rapidly until at  $N$  it becomes infinite, which agrees with the initial assumption that  $V_a = 0$ . It is to be noticed that the throat is reached when the pressure has dropped to about  $0.58 p_a$ . From this point on the nozzle flares indefinitely as long as the back pressure is dropped.

**Constant Heat Curves.**—The frequency with which the expression  $A \Delta \frac{V^2}{2g} = \Delta H + A(p_1 - p_2)\sigma$  must be evaluated in turbine design, and the inconvenience of solving for the final quality by equating the entropies in order to obtain  $H_2$  has made it advisable to plot total-energy curves,  $H + Ap\sigma = \text{constant}$ . In the range

of conditions usually met with the term  $Ap\sigma$  is negligible, so that the total-energy curves practically coincide with the constant heat curves. Care must be taken, however, in cases of extremely high pressures and wet steam to be sure that  $Ap\sigma$  is really negligible. Thus at the upper pressure quoted in Peabody's *Steam Tables* for hot water,

$$H = q = 402.2 \text{ B.T.U.} \quad \text{and} \quad Ap\sigma = \frac{336 \times 144 \times .016}{778} \\ = 0.995 \text{ B.T.U.},$$

or  $H$  differs from  $H + Ap\sigma$  by 0.25 per cent. approximately.

Thus we have, in general, the following relation existing between any two points of such a curve

$$H + Ap\sigma = q_1 + x_1 r_1 + Ap_1\sigma = q_2 + r_2 + c_p(T_s - T_2) + Ap_2\sigma,$$

while for all ordinary conditions reduces to

$$H = q_1 + x_1 r_1 = q_2 + r_2 + c_p(T_s - T_2).$$

To plot the curve in the saturated region a series of values for  $x$  from  $x = \frac{H - q}{r}$  must be computed for a sufficient number of different temperatures.

In the superheated region we must determine the points of intersection of the desired constant-heat curve with several constant-pressure curves (Fig. 39), by means of the relation

$$c_p(T_s - T_2) = H - (q_2 + r_2).$$

As  $c_p$  is a variable a few trials may be necessary before the correct value of  $T_s$  is obtained.  $T_s$  once known, the point can be at once located upon the corresponding constant-pressure curve.

The constant-heat curves once located on the dia-

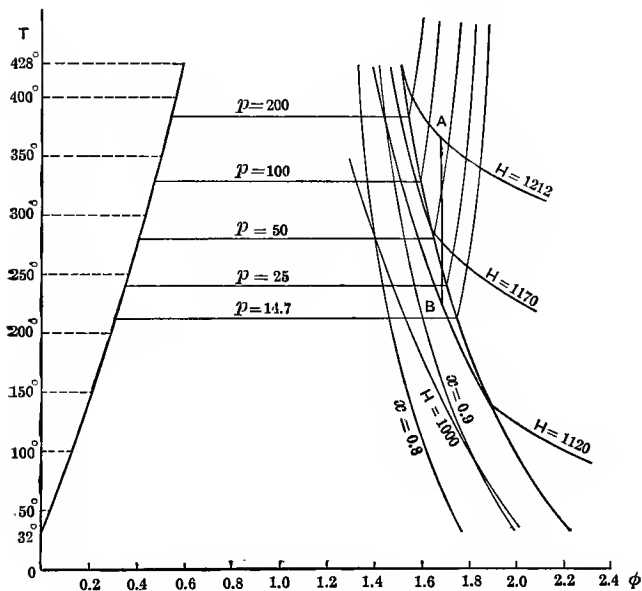


FIG. 39.

gram the solution of nozzle problems becomes very simple. Given any reversible adiabatic expansion, as

$AB$ , Fig. 39, the value of  $A4 \frac{V^2}{2g}$  is obtained by reading the values of  $H_A$  and  $H_B$  directly from the total heat curves.

**Peabody's Temperature-entropy Tables.**—It is at this point that the practical value and great convenience of Peabody's Entropy Tables become manifest. These tables cover that portion of the diagram (Fig. 40)

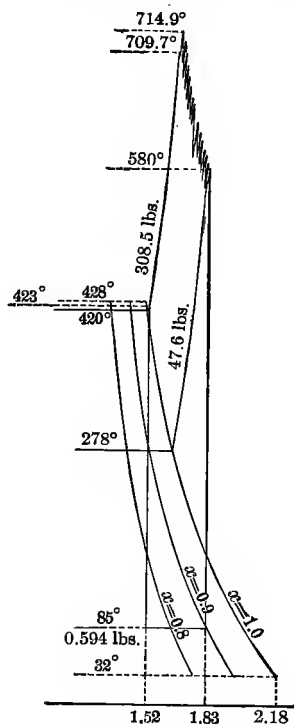


FIG. 40.

between  $\phi_1 = 1.52$  and  $\phi_2 = 1.83$  from the upper limits of our knowledge of saturated and superheated steam down to  $85^\circ$  F. In this region constant entropy lines are drawn for each 0.01 of a unit of entropy. These

lines are then crossed by a series of constant-pressure curves, so spaced that in the saturated region they coincide with isothermals spaced one degree apart from  $420^{\circ}$  to  $85^{\circ}$ . The table then tabulates for us along each constant-entropy line opposite each pressure curve the corresponding temperature of saturated steam, the quality (i.e., either the degrees superheat at constant pressure or the value of  $x$ ), the specific volume computed from the characteristic equation for superheated steam or from  $v = 0.016 + x(s - 0.016)$  as the case may be, and finally the value of the total heat  $H = q + r + c_p(T_s - T)$  or  $H = q + xr$ .

The entropy tables are then to be used in exactly the same manner as the  $T\phi$ -diagram. Each is entered by knowing the initial temperature and pressure, or temperature and quality. At this point the values of  $v$  and  $H$  are noted. The eye then runs down the constant-entropy line or column until the desired back pressure is found. At this point the quality, specific volume, and total heat are read.

**Design of Nozzle for Frictionless Adiabatic Flow.**—To illustrate the use of the  $T\phi$ -diagram or of the entropy tables let us find the throat and final diameter of a nozzle capable of delivering 10 H.P. net in kinetic energy at the final section. Assume the steam to be initially under 150 lbs. absolute per square inch and superheated  $100^{\circ}$  F., and that the discharge is at atmospheric pressure.

The nozzle must deliver  $10 \times 550 = 5500$  ft.-lbs. of kinetic energy per second. Assuming the steam to be initially at rest the kinetic energy delivered per pound of steam is

$$\begin{aligned} \frac{V^2}{2g} &= 778(H_1 - H_2) = 778(1249.6 - 1068.1) \\ &= 141,200 \text{ ft.-lbs. per second.} \end{aligned}$$

The amount of steam required per second is therefore

$$G = \frac{5500}{141,200} = 0.03895 \text{ lb.}$$

At the throat or minimum cross-section the pressure will have dropped to about 0.58 of its initial value or 87 lbs. At the throat, therefore, the specific volume will be 5.319 cu. ft. and the total heat 1199.7 B.T.U.

Hence

$$V_t = \sqrt{778 \times 64.32(1249.6 - 1200.4)} = 1569 \text{ ft. per sec.}$$

and

$$F_t = \frac{Gv_t}{V_t} = \frac{0.03895 \times 5.319}{1560} = 0.0001320 \text{ sq. ft.}$$

and

$$\text{Dia}_t = 0.1256 \text{ in.}$$

Similarly at the exit cross-section,

$$V_e = \sqrt{778 \times 64.32(1249.6 - 1068.1)} = 3014 \text{ ft. per sec.}$$

and

$$F_e = \frac{Gv_e}{V_e} = \frac{0.03895 \times 24.52}{3010} = 0.0003170 \text{ sq. ft.}$$

and

$$\text{Dia}_e = 0.2408 \text{ in.}$$

This computation, as well as the graphical method on pages 82–84, gives no idea as to the length of the nozzle. However, the exit cross-section would need to be placed at such a distance from the throat as to make the flare of the nozzle agree with the natural flare of the jet. The portion leading up to the throat must be rounded off into a smooth surface.

The design of a nozzle for an actual case, showing how to allow for friction losses, will be taken up later.

**Irreversible Adiabatic Processes.**—So far, in speaking of adiabatic lines, reference has been made only to reversible processes; that is, the expansion was frictionless and work was done at the expense of the internal energy either upon a piston or in imparting kinetic energy to the molecules of the expanding fluid. Suppose now that the adiabatic expansion occurs through a porous plug (as in Kelvin and Joule's experiments with gases) so arranged that as soon as velocity  $dV$  is developed, it is at once dissipated through friction into heat  $dQ$ , which is returned to the body at the lower pressure  $p - dp$ . The first operation is reversible and hence isentropic, the latter is equivalent to the addition of the heat  $dQ$  from some external source and hence the entropy increases by the amount  $\frac{dQ}{T}$ .

The actual operation, therefore, results in a drop of pressure and a growth of entropy without any increase

in velocity. Referring once more to the fundamental equation for the flow of fluids,

$$A \frac{V^2}{2g} = p_1 v_1 - p_2 v_2 + E_1 - E_2,$$

we obtain, since  $A \frac{V^2}{2g} = 0$  the result,

$$p_1 v_1 + E_1 = p_2 v_2 + E_2,$$

as the necessary relation between any two conditions of the fluid for such a completely irreversible process as flow without increase of velocity or the performance of outside work other than that required to crowd the substance into a new space.

**Irreversible Adiabatic Expansion of a Perfect Gas.—**  
In the case of perfect gas

$$pv + E = \text{constant}$$

reduces to

$$\frac{k}{k-1} \cdot pv = \text{constant, or simply } pv = \text{constant,}$$

i.e., the adiabatic process representing expansion with complete friction loss is at the same time an isothermal and an isodynamic change. An adiabatic process is, therefore, indeterminate unless specifically defined; if reversible it coincides with an isentrope; if absolutely irreversible, with a total energy curve which, in the case of perfect gases is also an isothermal; for all other



changes it occupies an intermediate position. (See pp. ix, x in the Introduction.)

Let  $AB$ , Fig. 41, represent such an irreversible adiabatic process. An entirely new interpretation must be given to the  $T\phi$ -diagram for such processes as this. The area under the curve  $AB$  no longer represents heat added from external sources (nor from any source), as no heat whatever has entered the body. An isothermal expansion of the gas has, however, occurred without

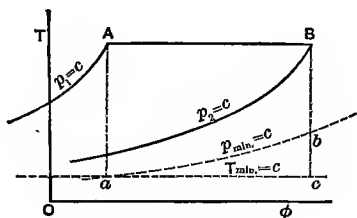


FIG. 41.

the performance of external work with the result that  $p_B < p_A$  and  $v_B > v_A$ .

The real significance of the change becomes apparent if we bear in mind Lord Kelvin's statement of the second law of thermodynamics that "it is impossible by means of inanimate material agency to derive mechanical effort from any portion of matter by cooling it below the temperature of the coldest of surrounding objects."

Let the dotted line (Figs. 41 and 42) represent the lowest available temperature, i.e., that of the atmos-

phere in gas-engine work. The minimum available pressure in actual work would likewise be that of the atmosphere.

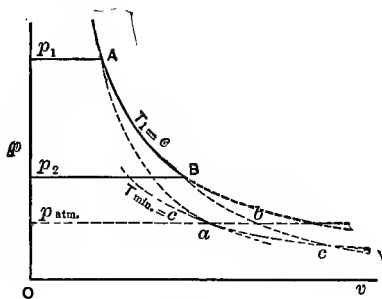


FIG. 42.

The work developed during isentropic expansion to atmospheric pressure would be

$$W_{Aa} = \frac{p_A v_A}{k-1} \left[ 1 - \left( \frac{p_a}{p_A} \right)^{\frac{k-1}{k}} \right]$$

and

$$W_{Bb} = \frac{p_B v_B}{k-1} \left[ 1 - \left( \frac{p_b}{p_B} \right)^{\frac{k-1}{k}} \right].$$

And since  $p_A v_A = p_B v_B$ ,  $p_a = p_b$  and  $p_A > p_B$  it follows that  $W_{Bb} < W_{Aa}$  by the amount

$$\frac{p_B v_B}{k-1} \left( \frac{p_b}{p_B} \right)^{\frac{k-1}{k}} - \frac{p_A v_A}{k-1} \cdot \left( \frac{p_a}{p_A} \right)^{\frac{k-1}{k}}.$$

Therefore

$$W_{Aa} - W_{Bb} = \frac{R}{k-1} T_1 \left[ \frac{T_b}{T_1} - \frac{T_a}{T_1} \right] = \frac{c_v}{A} (T_b - T_a).$$

Furthermore, while the work done upon the substance as it enters the cylinder or nozzle is the same ( $p_A v_A = p_B v_B$ ), the work required to exhaust it has increased by the amount

$$p_{\text{atm.}}(v_b - v_a) = R(T_b - T_a) = \frac{c_p - c_v}{A}(T_b - T_a).$$

The total loss in power resulting from the irreversible operation  $AB$  is therefore

$$\begin{aligned} \frac{c_v}{A}(T_b - T_a) + \frac{c_p - c_v}{A}(T_b - T_a) &= \frac{c_p}{A}(T_b - T_a) \\ &= \frac{c_p}{A} \cdot T_1 p_{\text{atm.}}^{\frac{k-1}{k}} \left( p_B^{\frac{1-k}{k}} - p_A^{\frac{1-k}{k}} \right). \end{aligned}$$

In other words, although the temperature was not changed and although no heat entered or left the body during the change,  $AB$ , nevertheless, because of it, heat to the amount  $c_p(T_b - T_a)$  has been made non-available for *actual* work.

Theoretically the loss in availability is not quite so great. It is possible to conceive of the expansion being carried below the back pressure until the lowest available temperature is reached, i.e., from  $b$  to  $c$ . Then on the return stroke the charge could be isothermally compressed from  $c$  to  $a$  so that the extra work  $bca$  could be gained and the corresponding heat loss avoided (see Fig. 41). The total heat made theoretically non-available would therefore be represented by the area under  $ac$ , i.e., by  $T_{\text{min.}}(\phi_B - \phi_A)$ .

Suppose as a further illustration that the operation  $AB$  were introduced into a cycle in which all the other operations were reversible isothermals and adiabatics (Fig. 43). The heat received from some outside source is shown by the area under the reversible isothermal  $CA$ . The heat rejected is that shown by the area under the reversible isothermal  $bc$ , while that rejected when the irreversible process  $AB$  is eliminated is shown by the area under  $ac$ . The heat exhausted during the cycle

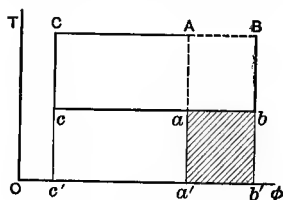


FIG. 43.

has thus been increased by the amount  $abb'a'$ , which is equal to the temperature of exhaust multiplied by the increase in entropy during the irreversible process  $AB$ . Swinburne generalizes this result in the following words: "The increase of entropy multiplied by the lowest temperature available gives the energy that either has been already irrevocably degraded into heat during the change in question, or must, at least, be degraded into heat in bringing the working substance back to the standard state. . . ." (See p. xiii of Introduction.)

**Transmission of Compressed Air Through Pipes.**—Air delivered by a compressor is heated above the tem-

perature of the atmosphere. This excess of temperature is soon lost by radiation in the pipe line and from that point on the flow is practically isothermal. There are, however, friction losses which result in a drop of pressure throughout the line. Both of these processes are irreversible and both produce loss of power—the first by a direct radiation of heat, the second by a re-

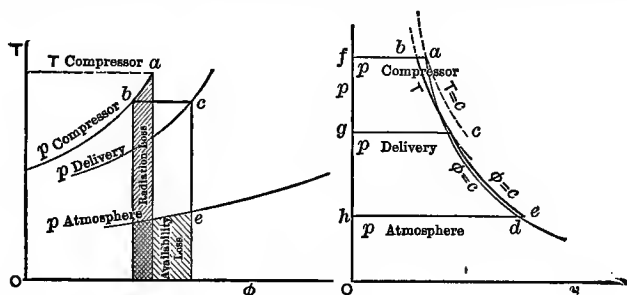


FIG. 44.

duction in availability of the energy remaining in the air.

In Fig. 44 let  $ab$  represent the cooling at practically constant pressure in the first part of the pipe line, and  $bc$  represent the irreversible adiabatic expansion caused by friction. If the air operates a motor it cannot be expanded below atmospheric pressure. Then  $adh$  represents the work which the air as delivered by the compressor could have developed during frictionless adiabatic expansion, while  $cehg$  represents the work which the air as finally delivered at the motor is capable of producing during frictionless adiabatic expansion.

The total loss of power is thus shown by the area *adecgfa*. A more complete analysis of this problem will be given in the chapter on air-compressors.

**Irreversible Adiabatic Expansion of Saturated and Superheated Vapors.**—In the case of saturated and superheated vapors the condition for completely irreversible flow  $pv + E = \text{constant}$  reduces to  $H + A p \sigma = \text{constant}$ . The irreversible adiabatic therefore coincides with the constant total-energy curves or approximately with the curve of constant total heat. Such a process occurs whenever steam pressure is lowered through a reducing valve, when the pressure drops as steam passes through the admission and exhaust ports of an engine, or when the pressure drops throughout the length of a pipe line. From its direct application in reducing valves this process is technically known as throttling and the constant total-energy curves are sometimes spoken of as throttling curves (*Drosselkurven*). Here, as in perfect gases, the adiabatic process is found to be indefinite unless the law of friction loss is specified. When there is no friction and part of the total energy goes into work or kinetic energy it coincides with an isentropic process. When the friction loss is complete so that no acceleration occurs it coincides with a throttling curve. In cases where the friction loss is only partial, as in turbine nozzles, it occupies some intermediate position.

It should be noticed that during throttling of saturated steam the moisture tends to evaporate and that

if the steam is nearly dry it may even become superheated during the process. We have found that for perfect gases the throttling curve represents an isothermal process; therefore, the further the throttling curves extend into the superheated region the more nearly horizontal they become, approaching the isothermal line as a limiting case.

**Loss of Availability due to Throttling of Steam.**—Suppose a pound of steam to have undergone an irre-

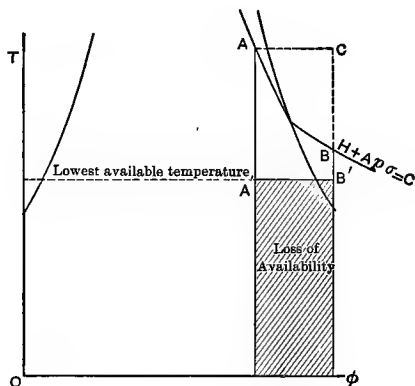


FIG. 45.

versible change of condition  $AB$ , Fig. 45. In order to restore the steam to its initial condition it must be compressed and heat must be rejected. If it were compressed adiabatically from  $B$  to  $C$  and then isothermally from  $C$  to  $A$  there would be rejected an amount of heat equal to the area under  $AC$ . Evidently less heat need be rejected if some path falling inside of  $BCA$

were utilized. Evidently,  $BB'A'A$  is the path along which the minimum amount of heat would be exhausted when  $B'A'$  represents the lowest available temperature. There would be no advantage in continuing the adiabatic expansion  $BB'$  below  $B'$ , because the steam could not be compressed isothermally at any lower temperature than  $T_{B'}$ , i.e., before heat could flow out it would need to be compressed back to  $B'$  again adiabatically. The heat which is unavoidably lost in restoring the steam from  $B$  to  $A$  is therefore equal to the lowest available temperature multiplied by the growth of entropy during the change  $AB$ .

*Problem 1.*—A throttling-valve reduces steam pressure from 150 lbs. gauge to 80 lbs. gauge. The steam initially contained 1 per cent. moisture. If the steam is used by an engine running at 2 lbs. absolute back pressure, find the loss per pound of steam caused by the valve.

We have (omitting  $Ap\sigma$ ),

$$q_{164.7} + 0.99r_{164.7} = q_{94.7} + r_{94.7} + c_p(t_s - t_{94.7}),$$

or

$$337.8 + 0.99 \times 856.9 = 294.4 + 890.7 + c_p(t_s - 323.9),$$

whence  $c_p(t_s - 323.9) = 1.0 \text{ B.}$

From the table of values of  $c_p$  we obtain 325.7 as the value of  $t_s$  which satisfies this equation.



Therefore,

$$\begin{aligned}\phi_1 &= \theta_1 + 0.99 \frac{r_1}{T_1} \\ &= 0.5235 + 0.99 \times 1.0381 \\ &= 1.5513\end{aligned}$$

$$\begin{aligned}\text{and } \phi_2 &= \theta_2 + \frac{r_2}{T_2} + c_p \log_e \frac{T_s}{T_2} \\ &= 0.4696 + 1.1369 + .56 \times 2.303 \times \log_{10} \frac{785}{783} \\ &= 1.6079.\end{aligned}$$

The temperature corresponding to 2 lbs. absolute is 126.2° F. The loss of available energy caused by the reducing-valve per pound of steam is therefore equal to

$$\begin{aligned}T(\phi_2 - \phi_1) &= (126.2 + 459.5)(1.6079 - 1.5513) \\ &= 33.2 \text{ B.T.U.}\end{aligned}$$

*Problem 2.*—A line of pipe delivers  $w_1$  pounds of steam per hour. By means of traps and a separator placed just above the throttle  $w_2$  pounds of water are removed per hour. The steam is thus practically dry at both ends of the pipe. The pressure drop from boiler to throttle is  $p_1 - p_2$  pounds. The steam-engine, using the  $w_1$  pounds, runs with  $p_3$  pounds back pressure. Find the total waste produced by transmission through the pipe.

Assume that the hot water is returned to the boiler

at the throttle temperature. The radiation loss (see Fig. 46) therefore equals

$$w_2(H_1 - q_2) + w_1(H_1 - H_2) \text{ B.T.U. per hour.}$$

Besides this direct loss of heat by radiation there is a further indirect loss due to the increase in entropy of the main body of steam. This represents the extra

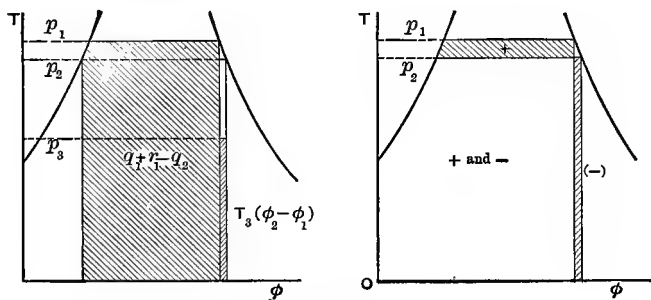


FIG. 46.

heat given up to the cooling water of the condenser during exhaust and equals

$$w_1 \cdot T_3(\phi_2 - \phi_1) \text{ B.T.U. per hour.}$$

The total loss due to pipe line is

$$w_2(H_1 - q_2) + w_1(H_1 - H_2) + w_1 \cdot T_3(\phi_2 - \phi_1) \text{ B.T.U. per hr.}$$

The heat supplied to the water in the boiler is

$$w_1(H_1 - q_3) + w_2(H_1 - q_2) \text{ B.T.U. per hour.}$$

The efficiency of transmission

$$= \frac{\text{Total heat supplied} - \text{total loss}}{\text{Total heat supplied}}.$$

**Peabody Calorimeter.**—The adiabatic expansion with increase of kinetic energy prevented by friction finds a very simple and valuable application in the Peabody throttling calorimeter. The calorimeter is a small expansion chamber connected to the steam main by means of a small pipe supplied with a valve. A second somewhat larger pipe, also containing a valve, exhausts the chamber to the atmosphere or any vacuum space. The whole instrument is heavily lagged to minimize the radiation and make the process adiabatic. The supply-pipe sometimes contains a standard orifice for measuring the steam passing through per hour. Steam-pressure gauges must be attached to the steam main and the calorimeter chambers. The latter must also contain a thermometer cup to permit of temperature readings. To operate the instrument the exhaust and admission valves are opened wide and the exhaust-valve afterwards so adjusted as to produce any suitable low pressure in the calorimeter. The instrument is ready for use after the readings become constant and the thermometer shows a minimum of about 10 degrees superheat. An excessive amount of superheat is not advisable.

The action, Fig. 47, is as follows: The steam expands

through the orifice with very little friction loss, the pressure dropping from  $b$  to about 0.58 of its initial value at  $a$ . The steam as it leaves the nozzle has thus acquired kinetic energy of approximately the amount  $(H_b - H_a) = H_{pb} - H_{.58pb}$ . The jet on entering the lower pressure of the calorimeter chamber expands in all directions, eddies are set up and the kinetic energy,

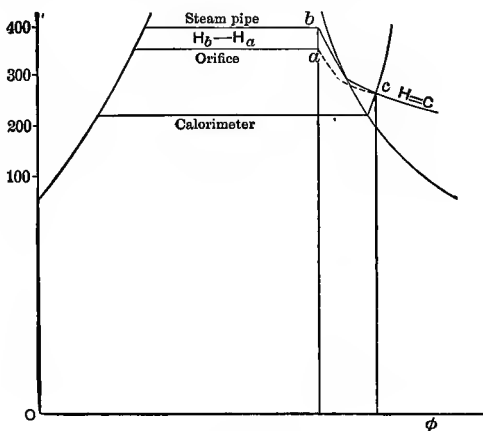


FIG. 47.

dissipated by friction, is restored to the steam as heat and thus serves to evaporate moisture. The cross-section of the calorimeter is so large that the velocity of the steam through it is practically the same as in the steam main. The ultimate change in kinetic energy is therefore practically zero, and as no external work is done and no heat lost or received the initial condition of the steam in the main and the final condition in the

calorimeter chamber must represent two points upon the same throttling-curve. The actual path of the steam, however, is not down the curve  $bc$  (Fig. 47), but more probably down some such path as  $bac$ , the part  $ac$  being indeterminate, as the steam is not in a homogeneous state. Between the initial and final conditions of the steam we have the simple relation

$$q_B + xr_B + Ap_B\sigma = q_c + r_c + c_p(T_s - T_c) + Ap_c\sigma,$$

whence

$$x = \frac{H_c - q_B - A(p_B - p_c)\sigma}{r_B},$$

or omitting  $A(p_B - p_c)\sigma$ ,

$$x = \frac{H_c - q_B}{r_B}.$$

The application of the Peabody calorimeter depends upon the possibility of superheating the steam by throttling. Thus suppose the minimum available pressure is that indicated in the diagram (Fig. 48), then steam at pressure  $p_B$  and of the quality 1, 2, 3, or 4 could have this quality determined by the calorimeter, because the throttling curves through these points intersect the curve  $pc=c$  in the superheated region, so that  $H_c = q_c + r_c + c_p(T_s - T_c)$  is known, while steam of the quality 5, 6, 7 could not have its quality determined, because the throttle curves through 5, 6, 7, . . . , intersect the lowest available pressure curve in the satu-

rated region, so that  $H_c = q_c + x_c r_c$  is unknown. The equation reads

$$q_B + x_B r_B = q_c + x_c r_c,$$

and as it contains two unknowns is indeterminate, so that the calorimeter does not give the desired information.

It is evident that by attaching the calorimeter to a vacuum its range of applicability may be increased

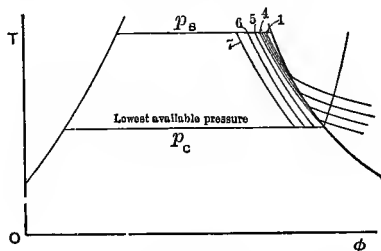


FIG. 48.

and further that the higher the steam pressure the greater the amount of moisture which the instrument can measure.

**Specific Heat of Superheated Steam.**—*a. By Throttling.*—The same equation which permits of the determination of the value of  $x$  by assuming  $c_p$  to be known may of course be used the other way around and starting with a known value of  $x$  permit the mean value of  $c_p$  to be calculated. This method was used by Grindley \*

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\* Trans. Royal Soc., vol. 194, sec. A, 1900.

and Griessman.\* Both started with what they considered to be dry steam, and permitted it to expand along a throttling curve into the superheated region, the one used an orifice in a glass plate, the other a porous plug of canvas washers surrounded by wood. Their results as well as those obtained by others † do not agree among themselves nor with those obtained in other ways.

The explanation of the discrepancy between the results of different investigators using the throttling method lies undoubtedly chiefly in the difficulty of knowing exactly the quality of the steam at the initial conditions.

The discrepancies between the results obtained by this method and other methods undoubtedly lies in the inaccuracies of the steam tables. Thus in the equation

$$q_1 + x_1 r_1 = q_2 + r_2 + c_p \cdot \Delta T_2,$$

or 
$$x_1 = \frac{q_2 + r_2 - q_1 + c_p \Delta T_2}{r_1},$$

$q_2 + r_2 - q_1$  ranges from 900–1200 B.T.U., so that an error of several B.T.U. in the determination of the various quantities and a large error in the assumption of the value of  $c_p$  would have but small effect upon the value of  $x_1$ . But in the equation

$$c_p = \frac{q_1 + x_1 r_1 - q_2 - r_2}{\Delta T},$$

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\* *Zeitschrift des Vereins Deutscher Ingenieure*, vol. 47, pp. 1852–1880.

† *Theses*, M. I. T., 1904, 1905, 1906.

the numerator is a small quantity and an error of a small amount which would not be appreciable in the individual terms produces a large error in the value of  $c_p$ . Furthermore, as  $r_1$  is in the neighborhood of 1000 B.T.U. a very slight error in the determination of  $x_1$  would produce a large percentage error in  $c_p$ .

It would seem therefore that this throttling method cannot give accurate results until the values in the steam tables have been more accurately determined and until the investigator can be absolutely sure that the steam is dry initially.

*b. By Superheating at Constant Pressure.*—A method used by Knoblauch and others, and which avoids the inaccuracies of the throttling method, is to pass steam already somewhat superheated through a bath electrically heated, thereby raising its temperature at constant pressure. If the electrical energy  $Q$  (corrected for the heat absorbed by the bath and metal of the calorimeter and that lost by radiation) required for heating  $w$  pounds and the corresponding increase in temperature  $T_2 - T_1$  are known, then the mean value of  $c_p$  for the given conditions is  $\frac{Q}{w(T_1 - T_2)}$ .

In actual operation the experiment is not entirely so simple as this. The steam in passing through the coil of pipe in the bath suffers a drop of pressure  $p_1 - p_2$  caused by the throttling action of the pipe friction. The friction loss occasions a drop of pressure  $T_1 - T_x$ ,



so that the heat added  $\frac{Q}{w}$  raises the temperature by the amount  $T_2 - T_x$  instead of only  $T_2 - T_1$ . Now the actual change is not along the broken path  $ACB$  (Fig. 49) but along some indeterminate path  $AB$ , therefore

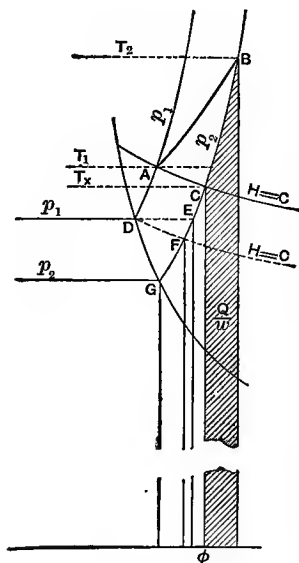


FIG. 49.

the throttling action does not all occur along the constant total-energy curve  $AC$ , but it is occurring along the whole series of throttling curves between  $A$  and  $B$  where the drop in temperature is not so great as along  $AC$ . It is therefore probable that the total drop of temperature caused by throttling is not  $T_1 - T_x$  but somewhat smaller than that. But if the drop of pressure  $p_1 - p_2$

is small and the increase in temperature  $T_2 - T_1$  large, the term  $T_1 - T_x$  is but a small correction factor, and a small error in it has but little effect upon  $c_p$ .

How is  $T_1 - T_x$  to be determined? The law of variation of  $c_p$  is unknown and to be investigated. It is known roughly that  $c_p$  varies along a constant-pressure curve, decreasing with increase of temperature. Therefore the mean value of  $c_p$  for  $T_1 - T_x$  is greater than the mean value for  $T_2 - T_1$ . It is next assumed that if the throttling curve  $DF$  is drawn that  $T_E - T_F = T_1 - T_x$ . This is not far from true as the slope of the curves is nearly the same. Now  $Q_{GF} = H_D - H_G$  and  $Q_{GE} = c_p(T_D - T_G)$ , whence

$$Q_{FE} = c_p(T_D - T_G) - (H_D - H_G),$$

and therefore

$$T_1 - T_x = T_E - T_F = T_D - T_G - \frac{H_D - H_G}{c_p}.$$

Introducing this correction in the original equation gives

$$c_p = \frac{Q}{w \left( T_2 - T_1 + T_D - T_C - \frac{H_D - H_G}{c_p} \right)},$$

$$\text{or} \quad w(T_2 - T_1 + T_D - T_C)c_p - w(H_D - H_G) = Q,$$

whence

$$c_p = \frac{\frac{Q}{w} + H_D - H_G}{T_2 - T_1 + T_D - T_C} = \frac{\frac{Q}{w} + H_D - H_G}{t_2 - t_1 + t_D - t_G},$$

or since  $H_D - H_G = 0.305(t_D - t_G)$ ,

$$c_p = \frac{\frac{Q}{w} + 0.305(t_D - t_G)}{t_2 - t_1 + t_D - t_G}.$$

Take as an illustration test No. 2 of the experiments of Knoblauch and Jakob,

$$p_1 = 29.7 \text{ lbs.}$$

$$p_2 = 27.7 \text{ lbs.}$$

$$t_D = 249.8$$

$$t_G = 245.8$$

$$t_1 = 329.9$$

$$t_2 = 411.6$$

$$w = 96.15 \text{ lbs.}$$

$$Q = 3758 \text{ B.T.U.}$$

$$\frac{Q}{w} = 39.08$$

$$c_p = \frac{39.08 + 0.305(249.8 - 245.8)}{411.6 - 329.9 + 249.8 - 245.8} = \frac{39.08 + 1.22}{81.7 + 4.0}$$

$$= \frac{40.3}{85.7} = 0.470.$$

It is true that the inaccuracies of Regnault's determination of the total heat of dry steam enter into the result, but whereas in the throttling method they affect the entire result, here they enter only as a minor variation in a small corrective term.

The apparent mean value of the specific heat as obtained from the direct observations is

$$c_p = \frac{39.08}{81.7} = 0.478.$$

which differs in this case by about 1.7 per cent. from the more probable value. It is probable that after

the first approximation of  $c_p$  has been determined for the entire range of experiments a second and more accurate determination of  $T_1 - T_x$  could be made, giving a final value of  $c_p$  as accurate as the original data.

**The Total Heat of Dry Saturated Steam.**—In discussing the throttling experiments of Grindley and Griessman it was shown that the discrepancies in the results were due to inaccuracies in the existing steam tables rather than in observational shortcomings. It remained for Dr. Harvey N. Davis\* of Harvard to bring order out of chaos and to point the way through a mass of apparently conflicting data and thus to lead to the establishment of new steam tables in which all the fundamental values, based upon different experiments, are found, when connected by Clapyron's equation

$$\frac{r}{T} = Au \cdot \frac{dp}{dT},$$

to check up with a maximum deviation not exceeding 0.2 of one per cent.†

Briefly, Dr. Davis proceeded as follows: The values of  $c_p$  obtained by Knoblauch and Jacob are taken as representing the best attainable data. These values are then substituted in the throttling observations of

\* Trans. A. S. M. E., 1908, pp. 741 to 774.

† Peabody, "Steam and Entropy Tables," p. 15.

Grindley, Griessman, and Peake\* and establish the *difference in value* between the total heats at two different temperatures, as  $H_1 - H_2 = c_p \cdot \Delta T_2$ . Thus suppose that observations of pressure and temperature have been taken at points  $A, B, C, D, E$  on the throt-

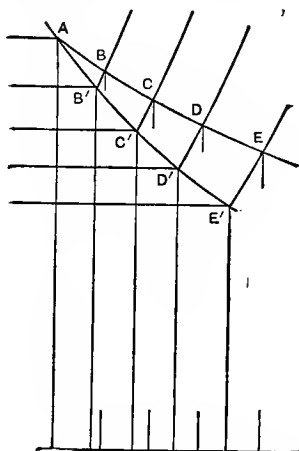


FIG. 49a.

ling curve  $AE$ , Fig. 49a. Then there exists between the points  $A, B', C', D'$ , and  $E'$  on the dry saturated curve these differences:

$$H_A - H_{B'} = c_p(T_B - T_{B'})$$

$$H_A - H_{C'} = c_p(T_C - T_{C'})$$

$$H_A - H_{D'} = c_p(T_D - T_{D'}), \dots$$

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\* Peake, Proc. Roy. Soc. (London), Series A, vol. 76 (1905), pp. 185-205.

The least accurate data in these, as with all other experiments with steam, are at the point *A*, due to the uncertainty as to its condition. The point *A* may, however, be eliminated from the observations by combining the above equations as follows:

$$\begin{aligned} H_{B'} - H_{C'} &= c_p(T_C - T_{C'}) - c_p(T_B - T_{B'}), \\ H_{B'} - H_{D'} &= c_p(T_D - T_{D'}) - c_p(T_B - T_{B'}), \\ H_{B'} - H_{E'} &= c_p(T_E - T_{E'}) - c_p(T_B - T_{B'}), \text{ etc.} \end{aligned}$$

Plotting  $\Delta H$  and  $T$  as rectangular coordinates, let the line  $H_B$  represent the total heat of the throttling

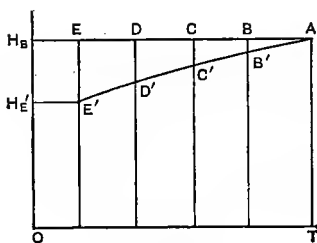


FIG. 49b.

curve  $AE$ , while the vertical lines  $BB'$ ,  $CC'$ ,  $DD'$ , etc., represent the heat of superheat at the respective conditions  $B$ ,  $C$ ,  $D$ ,  $E$ , and then the points  $B'$ ,  $C'$ ,  $D'$ , and  $E'$  will represent the total heat of dry steam at the respective temperatures. This avoids the necessity for the determination of the temperature of point  $A$ . (Fig. 49b.)

$B'$ ,  $C'$ ,  $D'$ ,  $E'$  thus represents a portion of the dry

saturated steam curve with the temperature of the points determined absolutely while the total heats are determined only relatively as the value of  $H_A$  and therefore of  $H_B$  is unknown. Grindley's data gave seven fragments of the saturated curve, Griessman's data eleven and Peake's six.

By skillful combination of these twenty-four fragments Davis obtained the curve showing the most probable relation between the total heat and the temperature of saturated steam from  $t=212^\circ$  F to  $t=400^\circ$  F.

This curve is represented within the limits of experimental accuracy by the second degree equation,

$$H = H_{212} + 0.3745(t - 212) + 0.000550(t - 212)^2.$$

For  $H_{212}$  Davis takes 1150.3 B.T.U., this being the mean of the values obtained by Henning\* and Joly†. Peabody, however, uses 1150 B.T.U., which is the result obtained from Henning's formula.

**Flow through a Nozzle.**—In the case of the flow through an actual nozzle the operation is not reversible. Heat is lost by radiation; heat is conducted through the metal of the nozzle from the higher to the lower temperatures; and friction occurs in varying amounts in different parts of the nozzle. The first loss, the rejection of heat as heat, *decreases* the entropy

\* Wied. Ann., IV, vol. 21 (1906), pp. 849–78.

† In an appendix (p. 322) to paper by Griffiths, Phil. Trans. (London), vol. 186 (1895), pp. 261 et seq.

of the fluid, while the other two losses both increase its entropy. It might happen that these opposing forces just balanced and then the expansion would be isentropic but not reversible. In general, however, the radiation loss may be made small, so that the operation is nearly adiabatic, but with increasing entropy

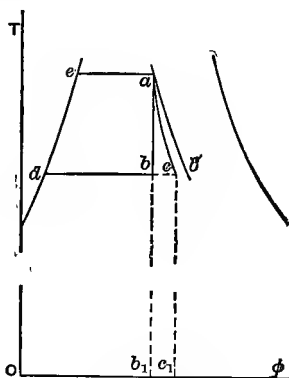


FIG. 50.

due to conduction along the nozzle and to friction losses.

Thus starting from  $a$ , Fig. 50, the actual expansion curve will lie between the isentrope  $ab$  and the constant-heat curve  $ab'$  in some such position as  $ac$ . The heat theoretically available is represented by  $abde$ . By friction, etc., the portion  $bcc_1b_1$  has been returned to the substance at a lower temperature, hence the kinetic energy of the jet at the exit  $c$  is equal to

$$\text{area } abde - \text{area } bcc_1b_1.$$



This loss would make itself noticeable in two ways. Decreased kinetic energy means decreased velocity, and increased entropy means increased volume. That is, if a nozzle were constructed from the dimensions necessary to give frictionless adiabatic flow and drilled at different points so as to measure the pressure, the observed pressure would be found to be greater at any given cross-section than the pressure for frictionless flow. Stodola's experiments show this, and also that the loss is at first slight, being practically negligible down to the throat, but increasing from there onward more and more rapidly as the velocity increases. That is, the curve  $ac$  would at first closely approximate  $ab$ , but lower down branch off more and more toward the right.

As soon as the curve  $ac$  has been accurately located it can be projected into the  $pv$ -plane and the corresponding areas for different cross-sections of the nozzle determined in the manner already indicated for the ideal case of frictionless flow.

**Adiabatic Expansion with Partial Friction Loss.**—The better to understand the phenomena of flow in a nozzle let us discuss the flow of steam in a non-conducting nozzle in which, however, there is the usual friction loss.

Let  $ab$  (Fig. 51) represent an isentropic expansion,  $ai$  a constant-energy expansion, and  $ac$  an adiabatic expansion with some friction loss, all from the same

initial condition  $a$ , at pressure  $p_1$ , down to the same back pressure  $p_2$ . At  $b$  the kinetic energy is greater than that at  $a$  by the amount  $abgk$ , at  $i$  the kinetic

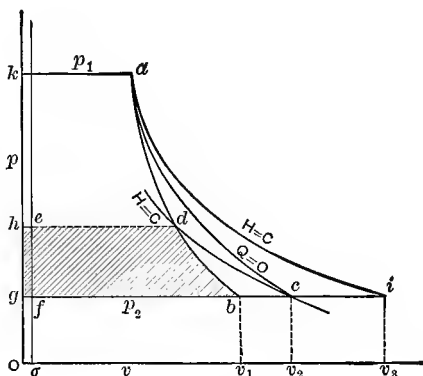


FIG. 51.

energy is the same as at  $a$ , and at  $c$  it possesses a value intermediate between that at  $b$  and  $i$ . Thus

$$A \cdot \frac{V_b^2 - V_a^2}{2g} = H_a - H_b = A \int_b^a v dp$$

and 
$$A \cdot \frac{V_c^2 - V_a^2}{2g} = H_a - H_c,$$

whence the loss of kinetic energy caused by the friction along  $ac$  is shown by the difference, or

$$A \cdot \frac{V_b^2 - V_c^2}{2g} = H_c - H_b.$$



would move more and more to the right, so that the utilized heat  $H_a - H_d$  would grow continually smaller, and finally in the limiting case where the friction loss is complete  $ac$  coincides with  $ai$  and  $H_a - H_d$  reduces to zero.

If we analyze closely such an irreversible adiabatic expansion as  $ac$ , we notice that it is made up of infinitely small transformations of heat into kinetic energy and of kinetic energy back into heat, and, taken in that sense, we see that the total area under  $ac$  can be interpreted to mean the heat equivalent of the friction loss. But we have already found that the loss of kinetic energy is shown by the area under  $bc$ , so the total friction loss exceeds the loss of kinetic energy by the amount represented by area  $abc$ . In other words, all of the friction loss is not irretrievable, because although the friction loss has occurred the energy has nevertheless been returned as heat at lower temperature, and is therefore still capable of being partially turned back into work. That is, at any intermediate point on the line  $ac$  the total heat is greater than that on the line  $ab$  for the corresponding pressure, and therefore more work is theoretically obtainable from it. The efficiency of the excess heat is, however, not as great at this lower temperature as it was originally, but its availability is not wholly destroyed until the expansion is carried down to the lowest possible temperature.

**Design of a Nozzle for Actual Flow.**—The design of

a nozzle for actual conditions is fundamentally the same as the case discussed on pp. 87, 88, except that the law of friction loss is now assumed known and provision made for the resulting decrease in velocity and increase in specific volume. Thus if the friction loss at any given section is  $f$  per cent. the available kinetic energy at that section is only  $1 - \frac{f}{100}$  parts of that available for frictionless flow, or

$$\frac{V_2^2}{2g} = 778 \left( 1 - \frac{f}{100} \right) (H_1 - H_2),$$

whence the actual velocity is

$$V_2 = \sqrt{2g \times 778 \left( 1 - \frac{f}{100} \right) (H_1 - H_2)} \text{ ft. per second.}$$

The kinetic energy lost by friction is restored as part of the total heat and results simply in improving the quality of the steam. Thus if the quality were  $x_2'$  as the result of isentropic expansion, the heat  $\frac{f(H_1 - H_2)}{100}$  would by further evaporation cause an increase in  $x_2$  of the amount,

$$\Delta x_2' = \frac{f(H_1 - H_2)}{100r_2},$$

so that the actual value of the quality would be

$$x_2 = x_2' + \frac{f(H_1 - H_2)}{100r_2}.$$

The actual specific volume would then be

$$v_2 = 0.016 + x_2(s_2 - 0.016).$$

In case the steam were superheated at the end of the isentropic expansion the heat  $\frac{f(H_1 - H_2)}{100}$  would produce an increase in superheat of the amount,

$$\Delta t_2' = \frac{f(H_1 - H_2)}{100c_p},$$

and the final temperature would be

$$t_2 = t_2' + \frac{f(H_1 - H_2)}{100c_p}.$$

The final volume  $v_2$  could then be found from the equation

$$pv = 85.85T - 0.256p,$$

or from the entropy tables.

If the weight passing the section per second is  $G$  pounds the cross-sectional area is given by

$$F_2 = \frac{Gv_2}{V_2}.$$

The value of  $G$  is determined by knowing the power which the nozzle must develop and the total friction loss at the exit section. Thus for  $n$  horse-power de-

livered by the jet and  $f_f$  as the total percentage friction loss at the final section,

$$\frac{V_f^2}{2g} = 778 \left( 1 - \frac{f_f}{100} \right) (H_1 - H_f),$$

and

$$G = \frac{n \times 550}{\frac{V_f^2}{2g}} = \frac{n \times 550}{778 \left( 1 - \frac{f_f}{100} \right) (H_1 - H_2)}.$$

*Problem.*—Find the throat and final diameters of a nozzle to develop 10 H.P. in the issuing jet, assuming a friction loss of 3 per cent. and 20 per cent. at these sections respectively. The steam is initially at 150 lbs. absolute pressure and superheated 100° F., and the back pressure is 4 lbs. absolute.

The actual kinetic energy of the issuing jet per pound is

$$\frac{V_f^2}{2g} = 778 \times 0.80 \times (1249.6 - 987.4) = 163,200 \text{ ft.-lbs.}$$

whence

$$V_f = \sqrt{64.32 \times 163,200} = 3240 \text{ ft. per second.}$$

and

$$G = \frac{10 \times 550}{163,200} = 0.03370 \text{ lbs. per second.}$$

or equals

$$\frac{0.03370 \times 3600}{10} = 12.13 \text{ lbs. per H.P. per hour.}$$

The value of  $x_f$  from the tables is 0.8617 and the increase in  $x_f'$  due to friction loss is

$$\Delta x_f' = \frac{0.20 \times 262.2}{1005.5} = 0.0522,$$

and the actual final quality is

$$x_f = 0.8617 + 0.0522 = 0.9139.$$

The final specific volume is

$$\begin{aligned} v_f &= 0.016 + 0.914(90.4 - .02) \\ &= 82.7 \text{ cu. ft.} \end{aligned}$$

The final cross-sectional area is therefore

$$\begin{aligned} F_f &= \frac{0.03370 \times 82.7}{3240} \times 144 \\ &= 0.1239 \text{ sq. ins.} \end{aligned}$$

The final diameter is therefore 0.3972 inch.

The theory of the flow of fluids shows that for isentropic expansion the pressure in the throat of the nozzle is about 0.54 of the initial pressure for superheated steam and about 0.58 of the initial pressure for saturated steam. In case the expansion passed from superheated to saturated steam before the throat was reached the only available method of determining the throat pressure would be to plot the ratio  $\frac{V_2}{v_2}$  for several values of  $p_2$  and thus determine the pressure corresponding to the maximum value of  $\frac{V_2}{v_2}$ .



In this problem the steam is still superheated at the throat, so that the throat pressure is about

$$0.54 \times 150 = 81 \text{ lbs. abs.}$$

The actual kinetic energy of the jet at the throat per pound is

$$\frac{V_t^2}{2g} = 778 \times 0.97(1249.6 - 1194.5) = 41590 \text{ ft.-lbs.}$$

whence

$$V_t = \sqrt{64.32 \times 41590} = 1635 \text{ ft. per second.}$$

The superheat at the end of isentropic expansion amounts to  $23.6^\circ \text{ F.}$  This is further increased by the friction loss by the amount

$$\Delta T_t' = \frac{0.03 \times 55.1}{c_p} = \frac{1.653}{c_p}.$$

To solve this we need to know the momentary value of  $c_p$ , but not having that we notice in the entropy tables that the total heat at this point increases 7.8 B.T.U. for  $14.7^\circ$  superheating. This gives

$$c_p = \frac{7.8}{14.7} = 0.531,$$

whence

$$\Delta T_t = \frac{1.653}{0.531} = 3.1^\circ \text{ F.}$$

The actual superheat at the throat = 26.6° F.

By cross interpolation in the tables the corresponding specific volume is found as

$$v_t = \frac{1.65}{7.8}(5.678 - 5.550) + 5.609 = 5.636 \text{ cu. ft.}$$

The cross-sectional area at the throat is

$$\begin{aligned} F_t &= \frac{0.03370 \times 5.636}{1635} \times 144 \\ &= 0.01673 \text{ sq. ins.} \end{aligned}$$

The throat diameter is therefore 0.1460 ins.

Assuming the flare between throat and exit to be a uniform taper of one in ten the length of the nozzle beyond the throat would be

$$10(0.3972 - 0.1460) = 2.51 \text{ inch.}$$

## CHAPTER VI.

### MOLLIER'S TOTAL ENERGY-ENTROPY DIAGRAM.

THE important role played by the total energy,  $i = H + Ap\sigma = E + pv$ , of steam in the discussion of the phenomena of flow led Mollier\* to construct a diagram using  $i$  and  $\phi$  as the coordinates. In that portion of the diagram  $abcd$  required for turbine nozzles  $Ap\sigma$  is negligible, so that  $i = H$  practically, and the diagram is thus sometimes called the "total heat-entropy diagram."

**Description.**—The general character of the plot and its relation to the  $T\phi$ -diagram are shown in Fig. 53, where for convenience in projection from the  $T\phi$ - into the  $i\phi$ -plot, or the reverse, the  $pT$ - and  $pi$ -quadrants are also given. Plot first,  $q + Ap\sigma$  and  $\theta$ , and  $q + r + Ap\sigma$  and  $\theta + \frac{r}{T}$ , to obtain the water ( $w$ ) and dry steam ( $s$ ) lines. The isothermals representing vaporization at constant pressure are next obtained by laying off the heat of the liquid and the total heat of dry steam,

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\* Neue Diagramme zur technischen Wärmelehre, von Prof. Dr. R. Mollier, Dresden. Zeitsch. d. Ver. Deutsch. Ing., Bd. 48 S. 271-274.

$H_w$  and  $H_s$ , for the desired pressure, as at  $a$  and  $b$  on the  $H$ -axis, finding the corresponding points on the

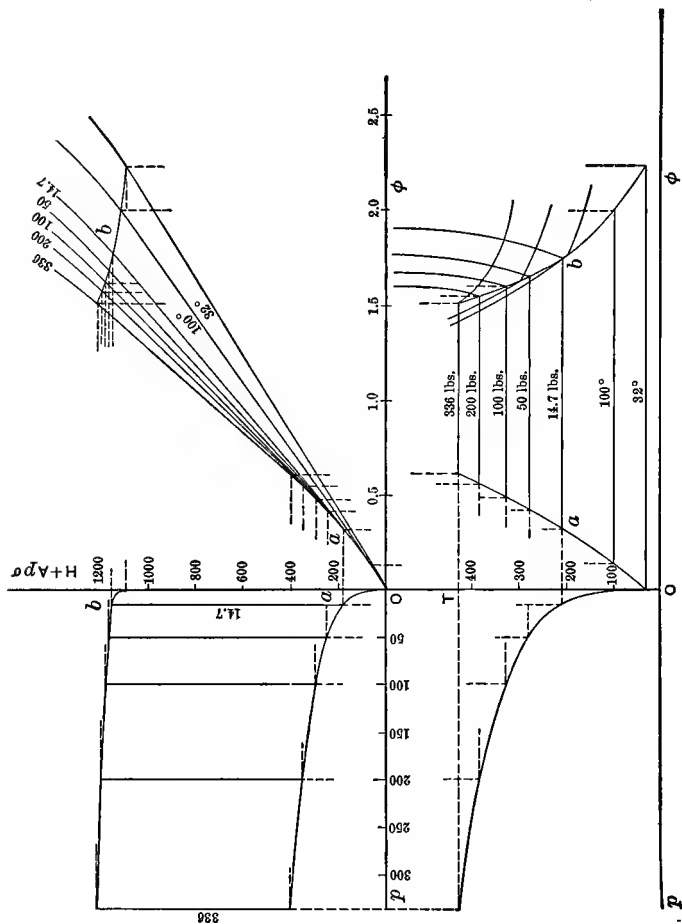


FIG. 53.

water and steam lines,  $a$  and  $b$ , and connecting these points by the straight line  $ab$ . The same line could

of course be determined graphically by projecting upwards the points  $a$  and  $b$  from the  $T\phi$ -plot. The "x-lines" or quality-lines are constructed by subdividing the constant-pressure lines between the water and steam lines into any desired number of equal parts, as tenths or hundredths, and connecting the corresponding divisions on the different lines by smooth curves.

To continue the constant-pressure curves beyond the dry-steam line into the superheated region we plot for each pressure the set of values

$$H = q + r + c_p(T_s - T) \quad \text{and} \quad \phi = \theta + \frac{r}{T} + c_p \log_e \frac{T_s}{T},$$

or, starting from the end of any pressure curve at the dry-steam line, lay off the extra values

$$\Delta H = c_p(T_s - T) \quad \text{and} \quad \Delta \phi = c_p \log_e \frac{T_s}{T}.$$

The accuracy of this part of the diagram is of course limited by our incomplete knowledge of the laws of variation of  $c_p$  for superheated steam. Thus the plots in Stodola's *Steam Turbines* are based upon Regnault's old value,  $c_p = 0.48$ , while those in Thomas' *Steam Turbines* are based upon the larger but still constant value,  $c_p = 0.58$ .

To plot the isothermals in the superheated region, start with the temperature corresponding to any pressure  $A$  on the dry-steam line, then the heat required to

superheat at constant pressure to this same temperature from any lower pressure will be  $c_p(T_B - T_c)$  and the point  $B$  may then be located on the curve  $p=c$  by laying off  $c_p(T_A - T_p)$  B.T.U. above the intersection of  $p=c$  with the dry-steam line (Fig. 54). The iso-

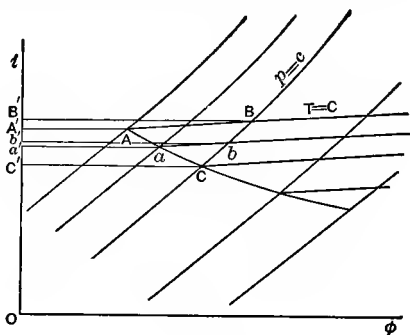


FIG. 54.

thermals, provided the  $T\phi$ -plane is already drawn, may be located graphically before the constant-pressure curves are drawn by projecting the value of  $\phi$  for each value of  $c_p(T_B - T_A)$  directly from the  $T\phi$ -plane.

**Reversible Adiabatic Processes.**—For frictionless adiabatic flow between the two pressures  $p_1$  and  $p_2$  the kinetic energy of a jet increases by the amount

$$A \cdot \frac{V_2^2 - V_1^2}{2g} = H_1 - H_2 + A(p_1 - p_2)\sigma$$

$$= i_1 - i_2.$$

Enter the plot at the point  $p_1 x_1$  or  $p_1 T_1$ , according as the steam is saturated or superheated, and follow down

the isentrope thus determined until the constant-pressure line  $p_2$  is reached. The vertical distance between the initial and final points gives at once  $i_1 - i_2$ , or the heat equivalent of the kinetic energy of the jet.

Assuming the initial velocity to be zero, the final velocity corresponding to any pressure, is given by

$$V_2 = \sqrt{\frac{2g}{A} \cdot \Delta i} = 223.7 \sqrt{\Delta i},$$

from which we obtain the following table:

$\Delta i$ (in B.T.U.)	$\sqrt{\Delta i}$	$\sqrt{\frac{2g}{A} \cdot \Delta i}$ (in ft. per sec.)
0.01	0.1	22.37
0.04	0.2	44.74
0.09	0.3	67.1
0.16	0.4	89.5
0.25	0.5	111.9
0.36	0.6	134.2
0.49	0.7	156.6
0.64	0.8	179.0
0.81	0.9	201.3
1.	1.	223.7
4.	2.	447.4
9.	3.	671.
25.	5.	1119.
64.	8.	1790.
100.	10.	2237.
144.	12.	2684.
225.	15.	3355.
324.	18.	4026.
400.	20.	4474.
441.	21.	4698.
484.	22.	4921.
625.	25.	5593.

An auxiliary plot (Fig. 55) using  $\Delta i$  as ordinates and  $V$  as abscissæ permits the determination of intermediate values, and forms a valuable addition not only to the  $i\phi$ -plot but to the entropy-tables as well.

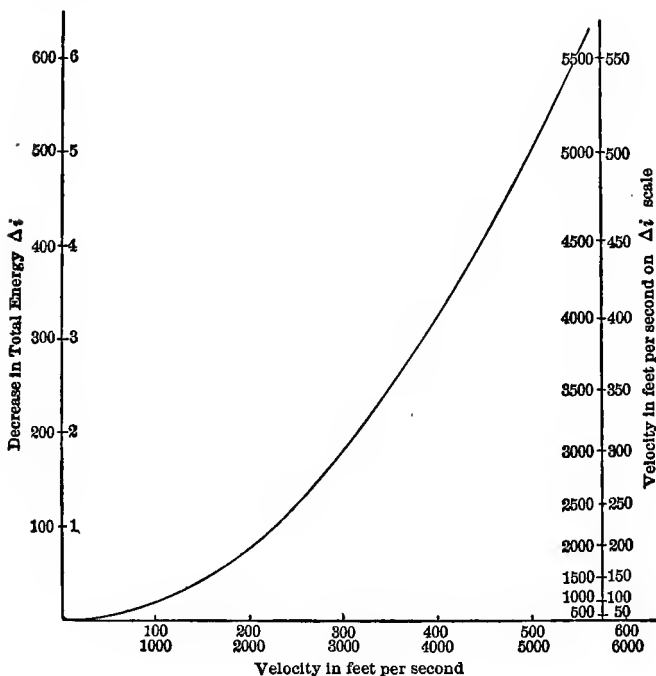


FIG. 55.

Two plots are advisable, drawn to different scales, one to be used for small values of  $\Delta i$ , the other for large values.

If we project the values of the velocity on to the  $\Delta i$ -axis, as indicated in Fig. 55, we obtain two scales,



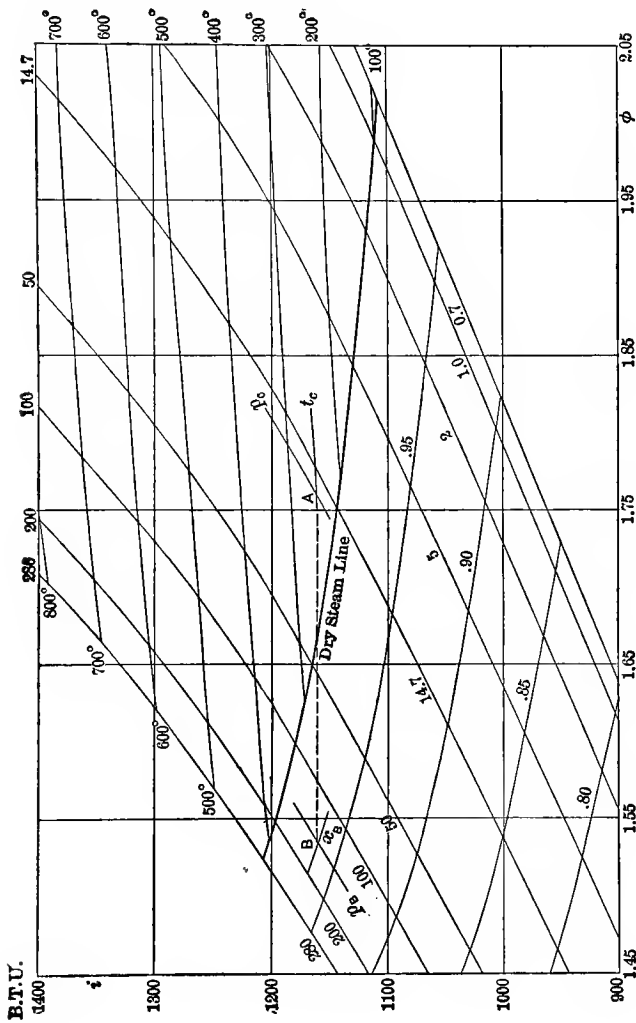


Fig. 56.

one showing  $4i$ , the other  $V$ , both measured from the same origin. If a sufficient number of intermediate values of  $V$  are indicated the scale may be plotted on a separate piece of paper, and can then be applied directly to the  $i\phi$ -diagram by placing the scale parallel to the isentropic curves with its zero at the initial point of the expansion. The position of the final pressure line on the scale will indicate the velocity directly in feet per second.

**Reduction of Pressure by Throttling.**—Throttling curves represent processes during which the total energy remains constant, and these are therefore represented by horizontal lines in the  $i\phi$ -plot. Such a plot permits of the ready determination of the quality of steam from the calorimeter observations. Thus if  $A$  (Fig. 56) is the point determined by the pressure  $p_c$  and temperature  $t_c$  in the calorimeter, proceed from  $A$  horizontally to the left until the curve  $i_A$  intersects the constant-pressure curve corresponding to the boiler pressure  $p_B$ , say at  $B$ . The position of  $B$  with reference to the  $x$  lines gives the desired quality  $x_B$ .

## CHAPTER VII.

### THE TEMPERATURE-ENTROPY DIAGRAM FOR MIXTURES (1) OF GASES, (2) OF GASES AND VAPORS, AND (3) OF VAPORS.

**Mixture of Perfect Gases.**—If several gases be mixed in the same vessel, the pressure of the mixture is equal to the sum of the pressures which the gases would exert if they occupied the whole space separately. This result discovered experimentally by Dalton is true of course only so long as the molecules do not sensibly obstruct each other. It may be assumed to hold rigidly in the ideal case of perfect gases.

For any given mixture undergoing reversible operations it must also be assumed that heat interchanges occur instantaneously, so that during any change whatever the pressure and temperature at any given instant are always uniform throughout. The common temperature may be measured directly, and if the weights of the different gases confined in the given space are known the specific volumes may be calculated. From these two quantities the respective specific pressures follow at once from the characteristic equations.

If the mixture is composed of  $n$  constituents possessing the weights  $m_1 \dots m_n$ , the specific pressures  $p_1 \dots p_n$ ,

the specific heats  $c_{v_1} \dots c_{v_n}$ , and  $c_{p_1} \dots c_{p_n}$ , respectively, the values of the specific heats of the mixture are given by

$$c_v = \frac{m_1 c_{v_1} + m_2 c_{v_2} + \dots + m_n c_{v_n}}{m_1 + m_2 + \dots + m_n},$$

and

$$c_p = \frac{m_1 c_{p_1} + m_2 c_{p_2} + \dots + m_n c_{p_n}}{m_1 + m_2 + \dots + m_n},$$

and the specific pressure by

$$p = p_1 + p_2 + \dots + p_n.$$

To apply the  $T\phi$ -analysis to any process occurring in such a mixture it is sufficient to treat it as a simple gas possessing the specific heats  $c_p$  and  $c_v$  and obeying the law  $p v = R T$ , where  $R$  is determined from some known values of temperature and volume. This is exemplified in gas-engine work, and even in air-compressor work: although air is always treated as a unit and its constituents never considered.

**Mixture of Gases and Vapors.**—Experiment has again shown where a given gas and liquid are chemically inactive and where the gas is not physically absorbed by the liquid, that when contained in the same vessel the liquid evaporates as if in a vacuum, and the pressure of the vapor is the same whether there is gas in the vessel or not. Common examples of such mixtures of interest to engineers are to be found in the air-pumps of steam-engine condensers and in compressors pumping moist air or those cooled by water injection.

Let there be  $w$  pounds of a gas per pound of saturated vapor in a given mixture and let it be desired to trace the relative changes between the two constituents as the mixture undergoes various definite changes.

From equation (8), page 15, we have as the change in entropy of  $w$  pounds of a perfect gas in going from any condition to any other condition the expression,

$$\Delta\phi_g = wc_p \log_e \frac{T_2}{T_1} - w(c_p - c_v) \log_e \frac{p_2}{p_1}.$$

Also the change in entropy between any two points in the region of saturated vapor is evidently equal to the difference in the total entropies of the two points, or

$$\Delta\phi_v = \theta_2 + \frac{x_2 r_2}{T_2} - \theta_1 - \frac{x_1 r_1}{T_1}.$$

Hence the total entropy change of the mixture is given by the sum

$$\begin{aligned} \Delta\phi_M = \theta_2 + \frac{x_2 r_2}{T_2} - \theta_1 - \frac{x_1 r_1}{T_1} + wc_p \log_e \frac{T_2}{T_1} \\ - w(c_p - c_v) \log_e \frac{p_{g_2}}{p_{g_1}}. \quad (1) \end{aligned}$$

A thermometer will give the common temperatures  $T_1$  and  $T_2$  and the aid of suitable tables (steam, ammonia, etc.) determines the values of the various heat quantities referring to the vapor. Subtracting from the gauge readings the vapor-pressures corresponding to

the observed temperatures from the tables leaves the gas pressures  $p_{g1}$  and  $p_{g2}$ .

There exist also the further relations between the volumes of the gas and vapor,

$$\sigma + x_1 u_1 = w v_{g1}$$

$$\sigma + x_2 u_2 = w v_{g2}$$

whence

$$x_1 = \frac{w v_{g1} - \sigma}{u_1}$$

$$x_2 = \frac{w v_{g2} - \sigma}{u_2}$$

$$\text{or} \quad \begin{aligned} & \frac{w \cdot \frac{RT_1}{p_{g1}} - \sigma}{u_1} \\ & \frac{w \cdot \frac{RT_2}{p_{g2}} - \sigma}{u_2} \end{aligned}$$

Substituting these values of  $x_1$  and  $x_2$  in the expression for  $\Delta\phi_M$  gives

$$\begin{aligned} \Delta\phi_M = & \theta_2 + \frac{r_2}{u_2 T_2} \left[ w \frac{RT_2}{p_{g2}} - \sigma \right] - \theta_1 - \frac{r_1}{u_1 T_1} \left[ w \frac{RT_1}{p_{g1}} - \sigma \right] \\ & + w c_p \log_e \frac{T_2}{T_1} - w (c_p - c_v) \log_e \frac{p_{g2}}{p_{g1}}. \end{aligned}$$

In this equation—provided it is possible to measure pressure and temperature—there are but two unknowns  $\Delta\phi_M$  and  $w$ . Hence if  $w$  be known  $\Delta\phi_M$  may be determined. For any given process it is sometimes possible to find the relative weights of gas and vapor. Let us now consider several special cases where the process is given.

(1) *Isothermal Change*.—During an isothermal process the pressure of the saturated vapor remains constant

but that of a gas will decrease as heat is added (expansion) so that the total pressure of the mixture will drop.

For such a change  $T_2 = T_1$ ,  $\theta_2 = \theta_1$ ,  $r_2 = r_1$ ,  $u_2 = u_1$ , so that the general expression for  $\Delta\phi_M$  reduces to

$$\Delta\phi_M = R w \frac{r}{u} \left[ \frac{1}{p_{g2}} - \frac{1}{p_{g1}} \right] - w(c_p - c_v) \log_e \frac{p_{g2}}{p_{g1}}.$$

To measure the heat received by the mixture add to the increase in total heat of the vapor the heat equivalent of the work performed by the gas, thus

$$Q = (x_2 - x_1)r + A w p_{g1} v_1 \log_e \frac{v_2}{v_1}.$$

Taking the values of  $x_2$  and  $x_1$  from page 142 we obtain

$$\begin{aligned} x_2 - x_1 &= \frac{w}{u_2} \cdot \frac{RT_2}{p_{g2}} - \frac{\sigma}{u_2} - \frac{w}{u_1} \cdot \frac{RT_1}{p_{g1}} + \frac{\sigma}{u_2} \\ &= RT \frac{w}{u} \left[ \frac{1}{p_{g2}} - \frac{1}{p_{g1}} \right], \end{aligned}$$

so that the equation for heat received may be written

$$Q = RT w \frac{r}{u} \left[ \frac{1}{p_{g2}} - \frac{1}{p_{g1}} \right] - A w R T \log_e \frac{p_{g2}}{p_{g1}}.$$

Of course this same expression can be obtained directly by multiplying  $\Delta\phi_M$  by  $T$ .

The work performed is given by

$$\begin{aligned} W_M &= \frac{A p_v u}{A} (x_2 - x_1) + w R T \log_e \frac{p_{g1}}{p_{g2}} \\ &= p_v (v_2 - v_1) + w R T \log_e \frac{v_2}{v_1} \\ &= R T p_v w \left[ \frac{1}{p_{g2}} - \frac{1}{p_{g1}} \right] + w R T \log_e \frac{p_{g1}}{p_{g2}}. \end{aligned}$$

Since the intrinsic energy of the gas does not change,

$$\Delta E_M = (x_2 - x_1) \rho = R T w \frac{\rho}{u} \left[ \frac{1}{p_{g2}} - \frac{1}{p_{g1}} \right].$$

As all the above expressions involve  $w$  it becomes necessary to determine the pounds of gas present in the mixture per pound of vapor before they can be evaluated.

Let  $a$  (Fig. 57) represent the initial state of the vapor and also of the mixture. Then in changing from  $a$  to  $b$  the vapor receives the heat shown by the area under  $ab$  and increases in volume from  $a'$  to  $b'$ , while the pressure maintains the constant value  $p_v$ . The heat received by the gas may be represented by some such area as that under  $bc$ , so that the total area under  $ac$  represents heat received by the mixture, while  $ab$ ,  $bc$ , and  $ac$  represent  $\Delta \phi_v$ ,  $\Delta \phi_g$ , and  $\Delta \phi_M$ , respectively. Lay off  $P_{M1}$  and  $P_{M2}$  equal to the initial and final gage readings, then  $Aa'$  and  $Bb'$  will represent the pressure



of the gas and  $AB$  referred to  $a'b'$  as the axis of volume will represent the  $pv$ -curve of the gas, while referred to  $Ov$  as the zero pressure line will represent the  $pv$ -curve of the mixture.

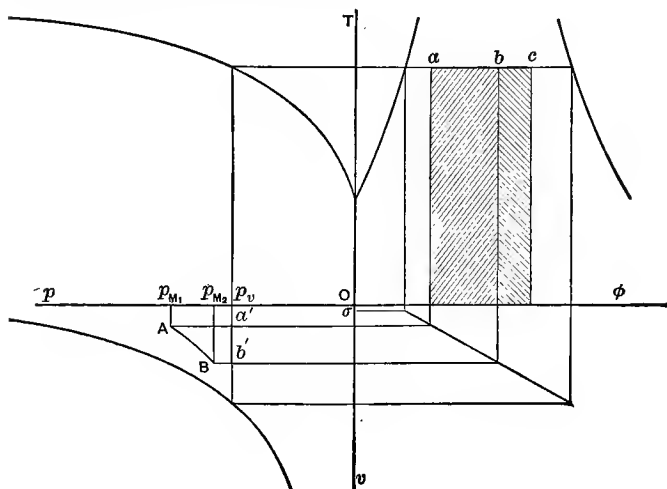


FIG. 57.

(2) *Heating or Cooling at Constant Volume.*—This introduces the special conditions,

$$x_1 u_1 + \sigma = x_2 u_2 + \sigma, \quad \text{and} \quad w v_{g_1} = w v_{g_2} = w \frac{RT_1}{p_{g_1}} = w \frac{RT_2}{p_{g_2}}.$$

By substitution of these values, the general expression for the entropy of a mixture reduces to

$$\Delta \phi_M = \theta_2 - \theta_1 + \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] [w \cdot v_g - \sigma] + w \cdot c_v \log_e \frac{T_2}{T_1}.$$

The heat received during this process is given by

$$\begin{aligned} Q &= q_2 + x_2 \rho_2 - q_1 - x_1 \rho_1 + w \cdot c_v (T_2 - T_1) \\ &= q_2 - q_1 + \left[ \frac{\rho_2}{u_2} - \frac{\rho_1}{u_1} \right] [w \cdot v_g - \sigma] + w \cdot c_v (T_2 - T_1). \end{aligned}$$

The change of internal energy is of course equal to the heat added.

Here again the value of  $w$  must be determined by suitable measurements before the expressions can be evaluated.

(3) *Isentropic Expansion or Compression*—If the mixture undergoes frictionless adiabatic expansion it is evident that this might occur in one of two ways: either (1) the entropies of both the vapor and the gas remain constant, or (2) if the entropy of the one varies a given amount that of the other varies an equal amount in the opposite direction. Which of these two possibilities is true and what are the character and magnitude of the actual change?

The fundamental relation now becomes, since  $\Delta \phi_M = 0$ ,

$$\begin{aligned} \theta_1 - \theta_2 + \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] \sigma \\ = w \left[ \frac{R r_2}{u_2 p_{g2}} - \frac{R r_1}{u_1 p_{g1}} + c_p \log_e \frac{T_2}{T_1} - A R \log_e \frac{p_{g2}}{p_{g1}} \right]. \end{aligned}$$

Provided the temperatures  $T_1$  and  $T_2$  may both be read this expression contains but one unknown,  $w$ , and thus gives directly the relative weights of gas and vapor

during such a change. Whether the work is performed upon a piston or in accelerating the mixture itself is non-essential.

$w$  being determined the quality of the vapor at any temperature  $T$  is given by

$$x = \frac{w \cdot \frac{RT}{p_g} - \sigma}{u}.$$

Having found  $w$  and the quality  $x_1$  it is now possible to determine the value of  $x$  at any number of tempera-

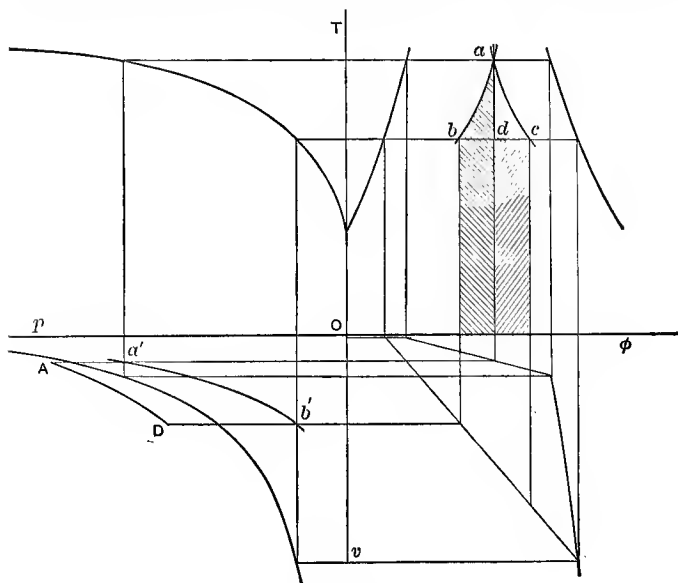


FIG. 58.

tures by taking suitable readings of pressure and temperature. Let  $ab$  (Fig. 58) be the  $T\phi$ -projection repre-

sending the locus of such a series of points for a pound of steam. Then  $ac$  must represent the corresponding curve for the  $w$  pounds of air, and  $ad$  will represent the isentropic path of the mixture. Thus the energy rejected by the steam as *heat* must be received by the gas as *heat* so that the process as regards its surroundings is adiabatic. If  $ac$  represent the path of the steam  $ab$  will represent that of the air.

Let  $AD$  represent the  $pv$ -curve of the mixture, then  $Aa'$ ,  $Db'$ , etc., will represent the pressure due to the air.

In order that  $\Delta\phi_v = \Delta\phi_g = 0$  it is necessary that

$$\theta_1 - \theta_2 + \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] \sigma = w \left[ \frac{R r_2}{u_2 p_{g2}} - \frac{R r_1}{u_1 p_{g1}} \right],$$

or that

$$w = \frac{\theta_1 - \theta_2 + \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] \sigma}{R \left[ \frac{r_2}{u_2 p_{g2}} - \frac{r_1}{u_1 p_{g1}} \right]}.$$

For all other values of  $w$ ,

$$\Delta\phi_v \gtrless 0,$$

and the magnitude may be found from

$$-\Delta\phi_v = \Delta\phi_g = w \left[ c_p \log_e \frac{T_2}{T_1} - AR \log_e \frac{p_{g2}}{p_{g1}} \right].$$

**The Determination of the Quality of Exhaust Steam.**

—In making the heat balance of an engine test, in a

Hirn's analysis, etc., it is often desirable to know the quality of the exhaust steam from a cylinder. Direct measurements are usually not possible with a throttling calorimeter due to the large quantity of moisture present. The above method lends itself to this purpose, provided

(1) An orifice can be inserted in the exhaust-pipe without appreciably changing the back pressure;

(2) That the friction loss down to the throat of the orifice is negligible, and

(3) That the pressures and temperatures in the exhaust-pipe and in the throat of the orifice can be accurately determined. This will necessitate the use of accurate manometers and of thermo-electric couples.

For the special case of steam and air we have

$$w = \frac{\theta_1 - \theta_2 + \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] \sigma}{53.35 \left[ \frac{r_2}{u_2 p_{g_2}} - \frac{r_1}{u_1 p_{g_1}} + 0.01025 l_{10} \frac{T_2}{T_1} - 0.00296 l_{10} \frac{p_{g_2}}{p_{g_1}} \right]}.$$

(4) *Constant-pressure Changes.*—These processes, although difficult to analyze, must receive attention, as in gas-turbine work it has been suggested that water be injected into gas burning at constant pressure, and although the steam eventually becomes superheated the mixture is initially one of saturated vapor and gas.

We have besides the general condition

$$\begin{aligned}\Delta\phi_M &= \Delta\phi_g + \Delta\phi_v = \theta_2 - \theta_1 \\ &+ wR \left[ \frac{r_2}{u_2 p_{g2}} - \frac{r_1}{u_1 p_{g1}} \right] - \sigma \left[ \frac{r_2}{u_2 T_2} - \frac{r_1}{u_1 T_1} \right] \\ &+ w \left[ c_p \log_e \frac{T_2}{T_1} - AR \log_e \frac{p_{g2}}{p_{g1}} \right];\end{aligned}$$

the further relation

$$p_M = p_g + p_v = \text{constant}.$$

$p_M$  may be constant provided both  $p_v$  and  $p_g$  are constant or if  $p_v$  increases as rapidly as  $p_g$  decreases. The first method is impossible because  $p_v$  cannot be

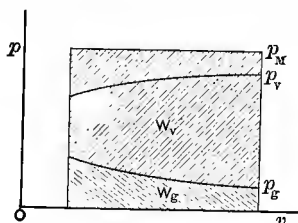


FIG. 59.

constant unless  $T$  remains constant, but if  $T$  is constant  $p_g$  decreases. The second solution is therefore the only possible one. That is, the  $p$ - $v$ -curves of the vapor and gas must be some such curves as  $p_v$  and  $p_g$  respectively in Fig. 59, where  $p_g + p_v = \text{constant}$ .

These curves fall under one of three headings:

- (1)  $p_v$  and  $p_g$  are both straight lines;
- (2)  $p_v$  is concave and  $p_g$  convex; or
- (3)  $p_v$  is convex and  $p_g$  concave.

The work developed by the mixture is of course  $p_M(v_2 - v_1)$ . The work developed by each constituent may be determined, provided temperature observations can be obtained at several volumes, as the simultaneous observations of  $p$ ,  $v$ , and  $T$  serve not only to determine the weight of gas present but also to define the curve  $p_g$ . The area under this curve represents that portion of the work performed by the gas and the area above the curve that performed by the vapor. If it is further possible to measure the vapor separately by weighing the liquid before it is fed into the combustion chamber or by condensation, the value of  $w$  is also known. It then becomes possible to find  $\Delta\phi_g$ ,  $\Delta\phi_v$ , and  $\Delta\phi_M$ .

The heat required for the change is given by

$$Q = A\Delta E + AW$$

$$= q_2 - q_1 + x_2\rho_2 - x_1\rho_1 + wc_v(T_2 - T_1) + Ap_M(v_2 - v_1),$$

where  $x_1$  and  $x_2$  are given by the formulæ on p. 142.

**Vapor-pressure of a Liquid Mixture.**—The pressure of the saturated vapor of a mixture of liquids was investigated by Regnault. The mixed vapors were found not to behave in general like a mixture of gases as regards pressure. Regnault distinguished three cases: (1) when the liquids do not mix, as water and benzene. In this case the vapor-pressure of the mixture is equal to the sum of the vapor-pressures of the constituents. (2) When the liquids mix partially or dissolve each other to a limited extent, like water and

ether. In this case the vapor-pressure of the mixture is less than the sum of the pressures of the constituents, or even less than one of them. Thus Regnault found

Temp.	Water-vapor Press.	Ether.	Mixture.
15.56° C.	13.16 mm.	361.4 mm.	362.95 mm.
33.08° C.	27.58 mm.	711.6 mm.	710.02 mm.

(3) The third case is that in which the liquids mix in all proportions. In this case the diminution of the vapor-pressure of the mixture is still more marked.

According to the experiments of Wüllner the vapor-pressure of any given mixture bears a constant ratio to the sum of the vapor-pressures of the constituents, at least when the liquids are mixed in nearly equal proportions. For other proportions this law is not quite exact. (Preston, *Theory of Heat*, p. 406).

**Mixtures of Liquids.**—Only that class in which the components exert their full individual pressures can be submitted to a general thermodynamic treatment. All other mixtures must be treated individually as special problems.

It is assumed that the mixture is of the same temperature throughout at any given moment and that the relative weights in a given space remain constant. Furthermore, that each substance fills the entire space.

**Heating at Constant Pressure.**—Suppose such a mixture of liquid confined in a cylinder under pressure and



heated from 32° F. until it begins to vaporize. The heat required would be the total heat of the liquids,

$$\begin{aligned} q_M &= w_1 q_1 + w_2 q_2 + w_3 q_3 + \dots = \Sigma w q \\ &= w_1 \int_{32}^t c_1 dt + w_2 \int_{32}^t c_2 dt + w_3 \int_{32}^t c_3 dt + \dots = \Sigma w \int_{32}^t c dt. \end{aligned}$$

The upper temperature, that is the temperature at which vaporization occurs, is determined by the relation,

$$p_M = p_1 + p_2 + p_3 + \dots$$

The entropy of the mixture is given by

$$\phi_{M_t} - \phi_{M_{32}} = w_1 \int_{32}^t \frac{c_1 dT}{T} + w_2 \int_{32}^t \frac{c_2 dT}{T} + \dots = \Sigma w \int_{32}^t \frac{c dT}{T}.$$

During vaporization

$$w_1 v_1 = w_2 v_2 = w_3 v_3 = \dots = \text{vol. of cylinder,}$$

so that the quality of each constituent can be found from the equations

$$x_1 = \frac{\frac{\text{vol.}}{w_1} - \sigma_1}{u_1}, \quad x_2 = \frac{\frac{\text{vol.}}{w_2} - \sigma_2}{u_2}, \dots$$

The heat required for vaporization is therefore equal to

$$(xr)_M = w_1 x_1 r_1 + w_2 x_2 r_2 + w_3 x_3 r_3 + \dots = \Sigma w x r.$$

The entropy is

$$\phi_{\text{vap.}} = w_1 \frac{x_1 r_1}{T_1} + w_2 \frac{x_2 r_2}{T_2} + w_3 \frac{x_3 r_3}{T_3} + \dots = \Sigma w x \frac{r}{T}.$$

The external work of vaporization is

$$AW_M = A(w_1 p_1 x_1 u_1 + w_2 p_2 x_2 u_2 + w_3 p_3 x_3 u_3 + \dots) \\ = \Sigma w \cdot x \cdot A p u$$

and the internal energy of vaporization is

$$\rho_M = w_1 x_1 \rho_1 + w_2 x_2 \rho_2 + w_3 x_3 \rho_3 + \dots = \Sigma w x \rho.$$

But each substance possesses at the temperature  $t$  its own specific volume, so that there is one value of  $wv$ , say  $w_1 v_1$ , which will be smaller than all the other values of  $wv$ . That is, this particular substance will be the first to have its liquid vaporized. At the moment this occurs

Vol. =  $w_1 s_1 = w_2(\sigma_2 + x_2 u_2) = w_3(\sigma_3 + x_3 u_3) = \dots$ ,  
whence

$$x_2 = \frac{\frac{w_1}{w_2} s_1 - \sigma_2}{u_2}, \quad x_3 = \frac{\frac{w_1}{w_3} s_1 - \sigma_3}{u_3}, \dots$$

$$x r_M = w_1 r_1 + w_2 x_2 r_2 + w_3 x_3 r_3 + \dots$$

$$\left(\frac{x r}{T}\right)_M = w_1 \frac{r_1}{T_1} + w_2 x_2 \frac{r_2}{T_2} + w_3 x_3 \frac{r_3}{T_3} + \dots$$

etc., etc.

Any further increase in volume at this temperature would mean superheating of  $w_1$  with consequent drop of pressure  $p_1$ , that is,  $p_M$  would decrease. This, however, is inconsistent with the assumption of constant pressure, so that the temperature would begin to rise

again in such a manner that the rise in pressure of  $p_2 + p_3 + p_4 + \dots$  would just counterbalance the drop in pressure of  $p_1$ . The increasing temperature causes  $s_1, s_2, s_3, \dots$  to decrease, and this, combined with the further movement of the piston, will cause one after another of the components to become superheated.

During this stage, when saturated and superheated vapors are intermingled, if we assume the total pressure, the temperature, and the total volume to be known, it is possible to determine for each substance whether or not

$$\frac{\text{vol.}}{w} \gtrless s.$$

Those that give  $\frac{\text{vol.}}{w} > s$  are superheated and the others saturated. The pressures of the superheated vapors may be found from their respective characteristic equations, the pressures of the saturated vapors from the vapor tables.

The total heat of such a mixture would no longer be equal to the sum of the total heats of its constituents, after the first one had begun to superheat, but could be found by adding to  $\Sigma wq$  the total work performed,  $Ap_M(\Sigma wv - \Sigma w\sigma)$ , and the increase of internal energy of each component.

The total entropy would be

$$\phi_M = \Sigma w \left( \theta + \frac{xr}{T} \right) + \Sigma w \left[ \theta_t + \left( \frac{r}{T} \right)_t + \int_t^{\theta_t} \frac{c_p dT}{T} \right].$$

After all the vapors were superheated the specific pressures of each could be found by inserting the known values of  $T$  and  $v$  in the various characteristic equations. Then there would exist as a check the relation,

$$p_M(\text{observed}) = (p_1 + p_2 + p_3 + \dots)_{(\text{computed})}.$$

This extra condition could, if necessary, be used to determine the weight of one of the constituents.

The entropy of the mixture of superheated vapors would be

$$\begin{aligned}\phi_M &= w_1 \left( \theta_1 + \frac{r_1}{T_1} + \int_{T_1}^T \frac{c_{p_1} dT}{T} \right) + w_2 \left( \theta_2 + \frac{r_2}{T_2} + \int_{T_2}^T \frac{c_{p_2} dT}{T} \right) + \dots \\ &= \Sigma w \left( \theta + \frac{r}{T} \right)_{T_{\text{sat.}}} + \int_{T_{\text{sat.}}}^T \frac{c_p dT}{T}.\end{aligned}$$

The heat required to raise the mixture at constant pressure from liquid at 32° F. to superheated vapors of temperature  $T$  is found by adding the increase of internal energy to the external work, or

$$\begin{aligned}H_M &= w_1 E_1 + w_2 E_2 + w_3 E_3 + \dots + A p_M (wv - \Sigma \sigma) \\ &= \Sigma w E + A p_M (wv - \Sigma \sigma).\end{aligned}$$

**Isothermal Expansion.**—Starting with a mixture of saturated vapors the quality of each vapor is given as before by

$$x_1 = \frac{\frac{\text{vol.}}{w_1} - \sigma_1}{u_1}, \quad x_2 = \frac{\frac{\text{vol.}}{w_2} - \sigma_2}{u_2}, \dots$$

and the pressure of the mixture by

$$p_M = p_1 + p_2 + p_3 + \dots$$

As heat is added, one after another of the constituents will become superheated, and as the temperature does not change, the pressure of such a superheated vapor will then decrease as the volume increases. The change of entropy can thus be computed separately for each component, so that

$$\begin{aligned} \phi_M = \Sigma \left[ \theta_2 + \frac{r_2}{T_2} + \int_{\text{sat.}}^{\text{sup}} \frac{c_p dT}{T} \right] \\ + \Sigma \left[ \theta_2 + \frac{x_2 r_2}{T_2} \right] - \Sigma \left[ \theta_1 + \frac{x_1 r_1}{T_1} \right], \end{aligned}$$

and the heat added during the change is

$$Q = T \cdot \Delta \phi_M.$$

The external work performed during such a change is

$$w = (Q - \Sigma \cdot \Delta E) \times 778.$$

## CHAPTER VIII.

### THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO HOT-AIR ENGINES.

THE Carnot cycle in the  $T\phi$ -plane is always a rectangle, but in the  $pv$ -plane its shape depends upon the nature of the working substance. For perfect gases the isothermals and frictionless adiabatics have nearly the same slope, so that to obtain an appreciable work area either the diameter of the cylinder or the length of the stroke must be made excessively large. That is, the excessive size and weight of the engine combined with large radiation and friction losses make the use of the Carnot cycle unfeasible in the case of hot air. Hence recourse has been had to certain of the isodiabatic cycles in the attempt to improve the work diagram.

The ideal cycle for the Stirling hot-air engine consists of the following events:

- (1) Heating at constant volume by passage of air through regenerator.
- (2) Expansion at constant temperature in contact with the hot surface of the furnace.

(3) Cooling at constant volume by return through the regenerator.

(4) Compression at constant temperature in contact with the cooling pipes.

The diagrams for such a cycle are shown in Fig. 60.

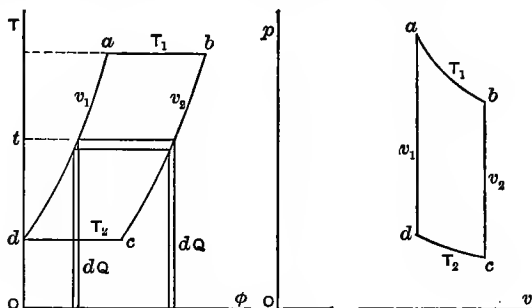


FIG. 60.

The criterion for such a cycle is that the heat rejected at any temperature  $T$  along  $bc$  shall equal that received at the same temperature along  $da$ . Hence

$$c_v dT + (c_p - c_v) T \frac{dv_1}{v_1} = dQ = c_v dT + (c_p - c_v) T \frac{dv_2}{v_2}.$$

As these equations both refer to the same isothermal  $t$  follows that

$$\frac{dv_1}{v_1} = \frac{dv_2}{v_2}, \quad \text{or} \quad \log v_1 + \log c_1 = \log v_2 + \log c_2,$$

whence  $c_1 v_1 = c_2 v_2$  or  $v_1 = c v_2$ .

That is, the lines  $ad$  and  $bc$  are "isodiabatic," as they satisfy the condition that the ratio of the volumes at the points of intersection with any isothermal is a constant.

The ideal cycle of the Ericsson engine is similar to that of the Stirling except that the heating and cooling occur at constant pressure instead of at constant volume.

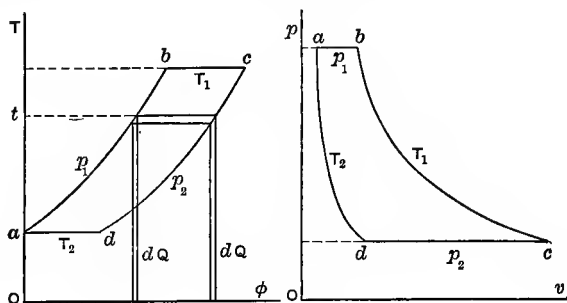


FIG. 61.

The ideal diagrams for such a cycle are shown in Fig. 61. In this case

$$c_p dT - (c_p - c_v) T \frac{dp_1}{p_1} = dQ = c_p dT - (c_p - c_v) T \frac{dp_2}{p_2},$$

whence

$$p_1 = c p_2.$$

Hence these curves are "isodiabatic," since the ratio of the pressures is a constant.

Both the Stirling and the Ericsson cycle give well-shaped indicator-cards and are thus better than the Carnot cycle mechanically.



**General Properties of Gas Cycles.**—It was shown in Chapter II that most changes of condition of gaseous mixtures can be represented as special cases of the polytrope  $pv^n = \text{const.}$ , where the specific heat of the change is defined by the equation  $c = c_v \cdot \frac{n-k}{n-1}$ . We may then consider the general gas cycle to consist of two pairs of such polytropic curves, whose specific heats are  $c_1$  and  $c_2$ , respectively, where  $c_2 > c_1$ .

Let  $abcd$  (Fig. 62) represent such a cycle. As  $\Delta\phi_{abc} = \Delta\phi_{cda}$  it follows that

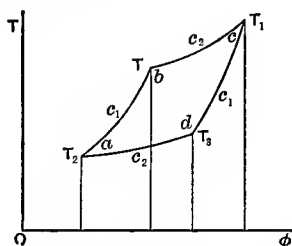


FIG. 62.

$$c_1 \log_e \frac{T}{T_2} + c_2 \log_e \frac{T_1}{T} = c_1 \log_e \frac{T_1}{T_3} + c_2 \log_e \frac{T_3}{T_2},$$

or 
$$c_1 \log_e \frac{TT_3}{T_1T_2} = c_2 \log_e \frac{TT_3}{T_1T_2}.$$

But  $c_2 > c_1$ , and therefore  $\log_e \frac{TT_3}{T_1T_2} = 0$ . There thus exists between the temperatures at the four corners of the cycle the simple relation,

$$TT_3 = T_1T_2.$$

Operating as a motor the work developed is equal to the difference between the heat received and the heat rejected, or

$$AW = c_1(T - T_2) + c_2(T_1 - T) - c_1(T_1 - T_3) - c_2(T_3 - T_2).$$

Eliminating  $T_3$  by means of the preceding relation, this reduces to

$$AW = \frac{c_2 - c_1}{T} (T_1 - T)(T - T_2).$$

The thermal efficiency is therefore

$$\eta = \frac{(c_2 - c_1)(T_1 - T)(T - T_2)}{T\{c_1(T - T_2) + c_2(T_1 - T)\}},$$

while its relative efficiency as compared with the Carnot cycle is

$$\frac{\eta_{c_1c_2}}{\eta_{\text{Carnot}}} = \frac{(c_2 - c_1)(T_1 - T)(T - T_2) \cdot T_1}{T\{c_1(T - T_2) + c_2(T_1 - T)\}(T_1 - T_2)}.$$

If the upper and lower temperatures are maintained constant it is evident that by considering point  $a$  to be fixed and  $c$  to move along the isothermal  $T_1$ ,  $T$  can be made to assume all intermediate values. During such a complete change the external work would pass from zero through a maximum back to zero.  $W$  is therefore a function of  $T$  and the value of  $T$  which will make  $W$  a maximum can be found by setting  $\frac{dW}{dT} = 0$ .

Thus

$$AW = (c_2 - c_1) \left( T_1 - \frac{T_1 T_2}{T} - T + T_2 \right);$$

$$\frac{dAW}{dT} = (c_2 - c_1) \left( 0 + \frac{T_1 T_2}{T^2} - 1 + 0 \right) = 0,$$

or

$$T = \sqrt{T_1 T_2} = T_3.$$

The substitution of this value of  $T$  in the expression for work gives

$$AW_{\max.} = (c_2 - c_1) \frac{(T_1 - \sqrt{T_1 T_2})(\sqrt{T_1 T_2} - T_2)}{\sqrt{T_1 T_2}}$$

$$= (c_2 - c_1)(\sqrt{T_1} - \sqrt{T_2})^2.$$

Under these conditions the heat received becomes

$$Q_1 = c_1(\sqrt{T_1 T_2} - T_2) + c_2(T_1 - \sqrt{T_1 T_2})$$

$$= (c_1 \sqrt{T_2} + c_2 \sqrt{T_1})(\sqrt{T_1} - \sqrt{T_2}),$$

and the thermal efficiency

$$\eta = \frac{(c_2 - c_1)(\sqrt{T_1} - \sqrt{T_2})^2}{(c_1 \sqrt{T_2} + c_2 \sqrt{T_1})(\sqrt{T_1} - \sqrt{T_2})} = \frac{(c_2 - c_1)(\sqrt{T_1} - \sqrt{T_2})}{c_1 \sqrt{T_2} + c_2 \sqrt{T_1}}.$$

The relative value of this cycle as compared with the Carnot is therefore

$$\frac{\eta_{c_1 c_2}}{\eta_{\text{Carnot}}} = \frac{(c_2 - c_1)T_1}{(c_1 \sqrt{T_2} + c_2 \sqrt{T_1})(\sqrt{T_1} + \sqrt{T_2})}.$$

*First Special Case.*—If  $c_2 = \infty$  the cycle consists of two isothermals and two polytropic curves, as shown

in Fig. 63. The expression for the maximum amount of work  $(c_2 - c_1)(\sqrt{T_1} - \sqrt{T_2})^2$  gives an infinite result, because no limitation has been placed upon the isothermal expansion. Although the general expressions possess indeterminate forms a special solution can be obtained directly from the  $T\phi$ -diagram.

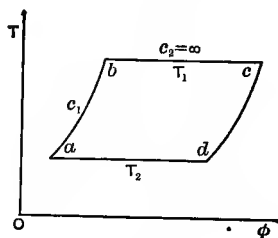


FIG. 63.

The work developed is equal to that developed in a Carnot cycle,

$$W = p_b v_b \log_e \frac{v_c}{v_b} \cdot \frac{T_1 - T_2}{T_1} = R(T_1 - T_2) \log_e \frac{v_c}{v_b},$$

or 
$$= R(T_1 - T_2) \log_e \frac{p_b}{p_c}.$$

The thermal efficiency of the cycle is therefore

$$\eta = \frac{AR(T_1 - T_2) \log_e \frac{p_b}{p_c}}{c_1(T_1 - T_2) + ART_1 \log_e \frac{p_b}{p_c}},$$

and its relative efficiency as compared with the Carnot cycle is

$$\frac{\eta_{c_1 \rightarrow \infty}}{\eta_{\text{Carnot.}}} = \frac{ART_1 \log_e \frac{p_b}{p_c}}{c_1(T_1 - T_2) + ART_1 \log_e \frac{p_b}{p_c}}.$$

(a) When  $c_1 = c_v$  this reduces to the Stirling cycle, consisting of two isothermals and two constant-volume curves. In this case it is more convenient to use the ratio of volumes  $\frac{v_c}{v_b}$  instead of the pressures  $\frac{p_b}{p_c}$ .

(b) When  $c_1 = c_p$  this reduces to the Ericsson cycle, consisting of two isothermals and two constant-pressure curves.

Of course the above discussion applies to the case when a regenerator is not used.

Other special cases will be considered in the Chapter on Gas-engines.

**The Non-regenerative Stirling and Ericsson Cycles.**— Inspection of the formula for efficiency or of Fig. 63 shows that the efficiency increases the smaller the relative magnitude of the heat  $c_1(T_1 - T_2)$  as compared with that received during the isothermal process. That is, the greater the range of pressure in the Ericsson cycle and the smaller the clearance in the Stirling cycle the higher the efficiencies. As the heat received during the isothermal change increases the efficiency of each cycle approaches that of the Carnot as a limit.

*Example.*—The small Ericsson pumping-engine (Stirling cycle without regenerator) has a piston 8 ins. in diameter and a stroke of 4 ins. The clearance is 200 per cent. of the piston displacement. Assuming the temperature of the fire to be 2500° F. abs. and that of the water-jacket 600° F., find the theoretical and relative efficiencies.

$$\begin{aligned}\eta_{\text{Stirling}} &= \frac{(T_1 - T_2)AR \log_e \left( \frac{\text{clearance} + \text{piston displacement}}{\text{clearance}} \right)}{c_v(T_1 - T_2) + ART_1 \log_e \left( \frac{\text{clearance} + \text{piston displacement}}{\text{clearance}} \right)} \\ &= \frac{(2500 - 600) \times \frac{53.35}{778} \times 2.303 l_{10} \frac{3}{2}}{0.169(2500 - 600) + \frac{53.35}{778} \times 2500 \times 2.303 l_{10} \frac{3}{2}} \\ &= 13.4 \text{ per cent.}\end{aligned}$$

$$\eta_{\text{Carnot}} = \frac{2500 - 600}{2500} = 76.0 \text{ per cent.}$$

$$\frac{\eta_{\text{Stirling}}}{\eta_{\text{Carnot}}} = 17.6 \text{ per cent.}$$

**The Ericsson Pumping-engine.**—The small Ericsson pumping-engine works but approximately upon the Sterling cycle, as the displacer and working piston are not timed so to operate that the different thermal events are entirely separate. This action, combined with the transference of heat between the air and the cylinder walls, results in a rounding off of the corners of the ideal cycle, giving an indicator card such as is shown in

Fig 64. It is to be further noticed that heating and cooling at constant volume exist only for a moment at the ends of the stroke. This is due to the almost entire absence of any regenerator. The only operation which in any way approximates that of a regenerator is the sudden transference of the air from one end of the cylinder to the other, causing it to move in a thin sheet through the narrow space between the displacer and the cylinder. It thus comes into contact with metal of varying temperatures, and is thus partly heated

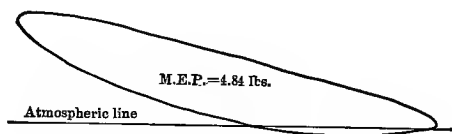


FIG 64.

or cooled during transmission, the rest of the heating or cooling occurs during the operation of the working piston.

This engine in itself is of no great interest to the engineer, but the  $T\phi$ -analysis of its indicator card is of value in that it permits of the direct application of the principles already discussed in the Chapter on Perfect Gases without introducing the various difficulties to be met with in gas-engine cards.

The same charge of air is used continuously in the Ericsson engine, the heat being received and rejected through the walls of the cylinder, and the indicator card

thus represents a closed cycle of a definite mass of air. It is true that during the movement of the displacer the air is partly in contact with both the heated and jacketed portions of the cylinder and is therefore not of uniform temperature, and similarly when the displacer is at rest, although most of the air is in contact with either the source or the refrigerator, part of it is nevertheless in the narrow space outside the displacer and thus at a different temperature from that of the main portion. There is thus at no instant a uniform temperature existing throughout the entire mass, but from the known pressure and volume the average temperature is determinable.

The range of both pressure and average temperature is so small that the air follows appreciably the laws of a perfect gas, so that  $c_p$  and  $c_v$  are constant.

The mass of air is unknown, so that only the ratio of the specific volumes and not their absolute values are known for different positions of the piston. The only item definitely known is therefore the pressure as measured from the indicator card. Therefore in the  $T\phi$ -projection the pressure will be the only property definitely known, but the ratio of the average temperatures is known for any two positions of the piston. Furthermore the indicated work being known the area of the indicator card in the  $T\phi$ -plane represents the heat equivalent of this work, and thus the B.T.U. per unit area are determinable. Evidently if the mean tem-



perature should be obtained experimentally for any position of the piston, the weight of air in the cylinder could be computed and thus the scales of temperature and entropy become known.

Fig. 65 shows the  $Tv$ - and  $T\phi$ -projections of the  $pv$ -card by the methods outlined in Chapter II. Because of the large clearance of the engine it was possible to economize space by placing the  $T\phi$ -projection to the left of the  $Tv$ -projection. As soon as one becomes familiarized with the methods the three diagrams can be superimposed without detracting any from their value.

The indicated power developed per cycle is obtained from the mean effective pressure and the dimensions of the engine. From this, together with the areas of the two diagrams, the foot-pounds and the B.T.U. per square inch may be determined in the  $pv$ - and  $T\phi$ -projections respectively.

The heat received and rejected by the air per stroke may now be measured as the surface scale of the  $T\phi$ -diagram is known, although the scale of the coordinates is unknown.

The heat theoretically available per cycle can be found in any given case by measuring the fuel and multiplying the weight per cycle by its calorific value. The difference between the heat available and the heat received represents the heat lost up the flue and to the surrounding atmosphere, cooling water, etc.

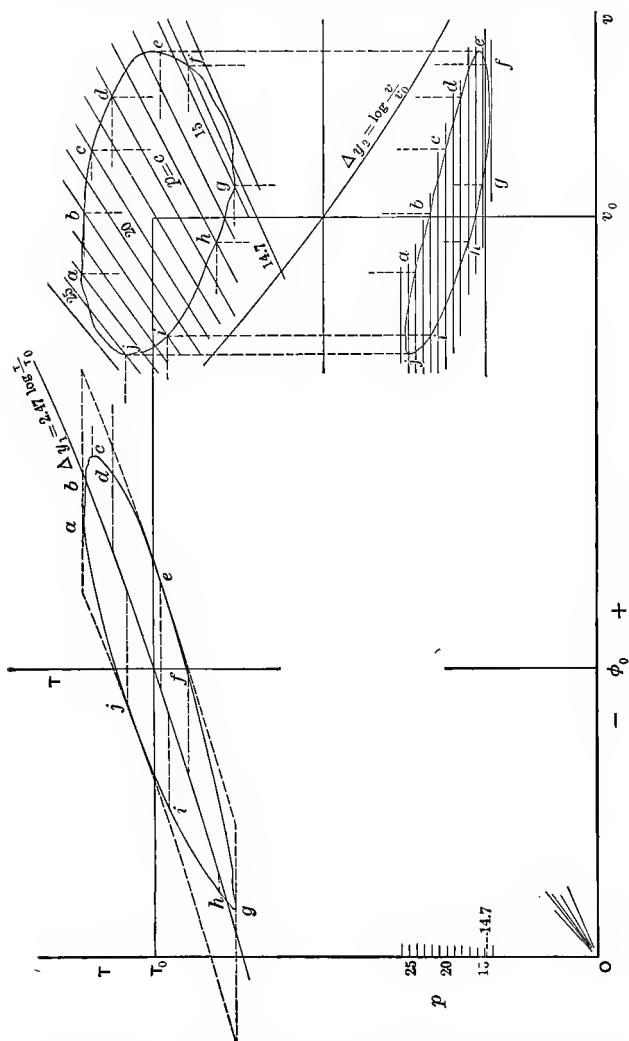


FIG. 65.

As a basis of comparison the Stirling cycle between the same temperature and volume limits is also given.

## CHAPTER IX.

### THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO GAS-ENGINE CYCLES.

No attempt will be made here to trace the heat losses due to radiation and the cooling water in the jackets, but the cylinder and piston will be considered impermeable to heat in all cases. Thus by a comparison of the ideal cards for different cycles the gain due to initial compression and the loss from incomplete expansion may be more clearly defined.

**The Lenoir, Otto, Atkinson, and Diesel Cycles.**—The Lenoir cycle was introduced in 1860. Its thermodynamic principles were retained in the different free-piston engines. These were uneconomical and noisy and have disappeared. The only remaining example is the Bischoff, a simple small vertical engine.

The Lenoir cycle consists of the following events:

(1) During the first part of the forward stroke a fresh explosive mixture is drawn in by the piston ( $aA$  in Fig. 66).

(2) A little before half-stroke is reached the supply-valve closes and the explosion occurs. In reality the

combustion requires an appreciable time for its completion, and thus the heating takes place while the piston is moving forward, i.e., at increasing volume, but for the ideal case the explosion will be considered instantaneous, and hence the heating will be at constant volume; along the line  $AB$ .

(3) The rest of the stroke represents adiabatic expan-

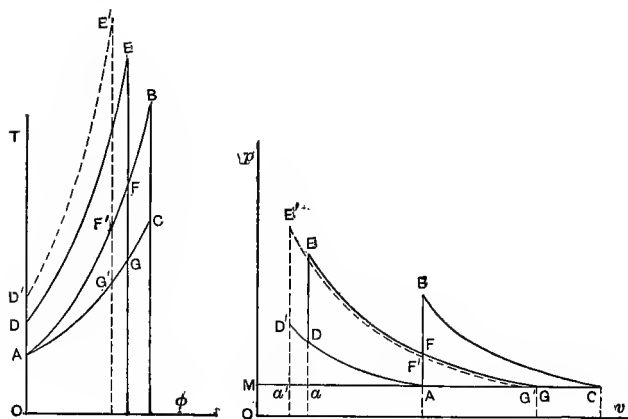


FIG. 66.

sion of the heated gas down to initial pressure; along  $BC$ .

(4) The return stroke, during which the products of combustion are exhausted. Thermodynamically this is equivalent to cooling at constant pressure; along  $CA$ .

About the time Otto and Langen were experimenting with the free-piston engine, Beau de Rochas described a cycle which would make possible the economical

running of a gas-engine. This was embodied by Dr. Otto in his silent engine in 1876 and has thus become associated, although wrongly, with his name.

The "Otto" cycle consists of the following events:

(1) The drawing into the cylinder at atmospheric pressure of a new explosive mixture throughout one complete stroke ( $aA$  in Fig. 66). The volume of the charge is  $MA$  and consists of the burnt products in the clearance space  $Ma$  from the last charge plus the fresh charge.

(2) The adiabatic compression of this charge on the return stroke of the piston  $AD$ . This compression of the gas into the clearance space is done at the expense of the energy in the fly-wheel.

(3) The ignition and explosion of the charge while the piston is at rest at the dead-centre, thus increasing the pressure and temperature at constant volume; along  $DE$ . Assuming that the same quantity of mixture is used by both the Lenoir and Otto engine, the heat generated by the explosion will be the same in both cases, i.e., the areas under the curves  $AB$  and  $DE$  are equal in the  $T\phi$ -diagram.

(4) The expansion of the heated gases throughout the entire stroke, assumed adiabatic; along  $EF$ .

(5) The drop in pressure due to the opening of the exhaust-valve while the piston is at the end of the stroke. This is equivalent to cooling at constant volume; along  $FA$ .

(6) The exhaust of the burnt gases during the return stroke  $Aa$ . Changes of location are not recorded in the  $T\phi$ -diagram.

In the Atkinson engine, now no longer made, the cycle was the same as the Otto up to the point  $F$ , and then, instead of releasing the hot gas, the expansion stroke was lengthened by means of an ingenious mechanism permitting the adiabatic expansion down to back pressure, as represented by  $FG$ . Then the exhaust stroke was from  $G$  to  $A$ , which thermodynamically is equivalent to cooling at constant pressure.

Comparing the Atkinson and Otto cycles it is at once evident that there is a loss of work and of heat equal to  $AFG$  in the  $pv$ - and  $T\phi$ -planes, respectively, due to incomplete expansion.

A comparison of the Atkinson and Lenoir cycles shows that as the heat received in both is the same while that rejected by the Lenoir engine is the greater (compare areas under  $CA$  and  $GA$ ), the efficiency of the Atkinson is the greater.

Theoretically, then, the Atkinson engine has the most perfect cycle of the three, but nevertheless it has been entirely superseded by the Otto engine. The reason for this becomes at once apparent from the diagram. Even if the area  $AFG$ , rejected by the Otto engine, due to incomplete expansion, were just equal to the area under  $GC$  of the Lenoir exhaust stroke, the Otto

would still be preferable to the latter because the same amount of power could be developed with a smaller engine. Suppose, now, the clearance space in the above Otto engine were decreased, so that the adiabatic compression would heat the gas to a higher initial temperature, as  $AD'$ . The explosion would now occur along the constant-volume curve  $D'E'$ , where the area under  $D'E'$  is equal to that under  $DE$ , as the same heat is generated in both cases. The adiabatic expansion would now be down  $E'F'$  and the exhaust would be along  $F'A$ . Hence the same Otto engine with increased initial compression due to decreased clearance would give increased efficiency, as the heat rejected under  $F'A$  is less than that rejected under  $FA$ . This engine would now be better than the Lenoir, both mechanically and thermodynamically. Furthermore the loss due to incomplete expansion becomes less because the heat thus rejected is reduced from  $AFG$  to  $AF'G'$ . That is, the higher the initial compression the less the theoretical superiority of the Atkinson over the Otto engine. And in the actual engine the increased complexity, size, friction loss, and danger of the Atkinson more than counterbalanced the theoretical superiority. Hence the Otto engine is practically the most efficient of the three.

In the Otto cycle the temperature at the end of compression is not very high relatively, so that during the first part of the combustion the working fluid is

much colder than the source of heat and does not attain this high temperature until the end of the combustion is reached. The heat is thus received at constant volume and increasing temperature instead of at constant temperature and increasing volume. It was shown in the first chapter that any such deviation from a Carnot cycle means a drop in efficiency. This led Diesel to invent a cycle during which most of the heat should be received at the highest available temperature. His method is to compress the air initially up to about five hundred pounds pressure to the square inch, so that its temperature is above the ignition-point of the combustible to be used. The injection of a small quantity of fuel causes the temperature to increase still further at constant volume up to that of the combustion; then as the piston moves forward the temperature of the gas is maintained nearly constant by the injection and combustion of further fuel. This lasts for about one-tenth of the stroke. The indicator-cards taken from such a motor show that the desired regulation is not perfect, the temperature sometimes rising, sometimes falling. This, however, only affects the magnitude of the gain from such a cycle, as all of the heat generated on the forward stroke is transmitted to the working fluid at an efficiency corresponding to that of the upper part of the Otto cycle. The cards show also that the expansion may or may not be carried down to the back pressure.



Fig. 67 shows the ideal diagrams for the Otto, Atkinson, and Diesel cycles for the same quantity of heat; that is, the areas under  $bc$  and  $b'gc'$  are the same in the  $T\phi$ -plane. The change from  $b$  to  $b'$  shows the increased compression in the Diesel motor,  $b'$  being at or above the temperature of ignition. The heat received

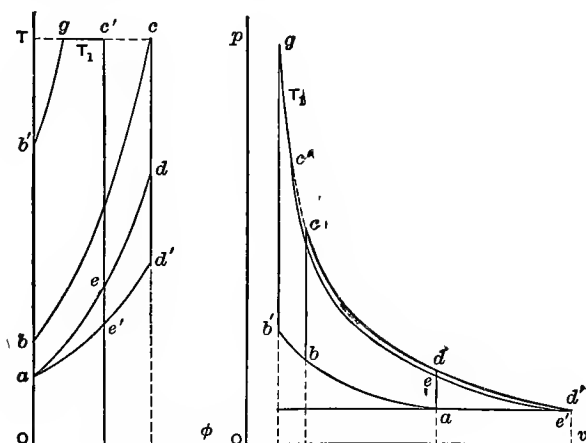


FIG. 67.

along  $b'g$  is not received under the most efficient conditions, but still with an efficiency equal to that of the best part of the Otto cycle, while that received along the "isothermal combustion line"  $gc'$  is obtained under conditions of maximum efficiency. The effect, as is clearly shown by the diagram, is to increase the amount of heat changed into work and to diminish the heat rejected. On the return stroke the conditions of the Carnot cycle are more closely approximated in

the Atkinson and Diesel cycles where the heat is rejected at constant pressure than in the Otto cycle where it is rejected at constant volume, as lines of constant pressure deviate from isothermals less than do lines of constant volume.

**The Brayton or Joule Cycle.**—Up to the present time the Otto cycle has been almost exclusively employed

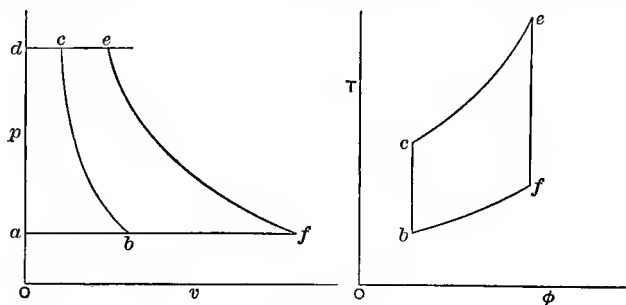


FIG. 68.

because of its ease of application in the reciprocating type of motor, but occasional attempts have also been made to heat the fuel under constant pressure. Such a cycle is now assuming importance as offering the most probable solution of the gas-turbine problem. Proposed by Joule this cycle is perhaps better known through its application in the Brayton engine. It consists of the following events (Fig. 68):

- (1) The charging stroke of the compressor cylinder, *ab*.
- (2) Adiabatic compression in the compressor, *bc*.

(3) Discharge from the compressor at constant pressure,  $cd$ .

(4) During the transfer from the pump to the working cylinder (or to the nozzle of the turbine) the charge receives heat either from external sources or from combustion under constant pressure, but with increasing volume and temperature.

(5) Admission stroke of the working cylinder,  $de$ . Processes (3), (4), and (5) occur simultaneously.

(6) Adiabatic expansion in the working cylinder or in the nozzle,  $ef$ .

(7) Discharge from the working cylinder against a constant back pressure,  $fa$ .

The net result is thus represented by the cycle  $bcef$ , consisting of two isentropic and two constant-pressure curves.

It is at once evident that the efficiency of the Brayton cycle, like that of the Otto, increases with increased initial compression.

It is instructive to compare the three ideal cycles of Carnot, Otto, and Brayton—involving as they do heating at constant temperature, constant volume, and constant pressure—under different conditions.

Let  $abcd$ ,  $abc'd'$ , and  $abc''d''$  (Fig. 69) represent the Otto, Brayton, and Carnot cycles respectively, for the same weight of charge working with the same initial compression. The areas under  $bc$ ,  $bc'$ , and  $bc''$  will then be equal.

The thermal efficiencies of the cycles are found as follows:

$$\eta_{\text{Otto}} = \frac{c_v(T_c - T_b) - c_v(T_d - T_a)}{c_v(T_c - T_b)} = 1 - \frac{T_d - T_a}{T_c - T_b}$$

$$= 1 - \frac{T_a}{T_b}, \quad \text{or} \quad 1 - \left(\frac{v_b}{v_a}\right)^{k-1}$$

$$\eta_{\text{Brayton}} = \frac{c_p(T_c - T_b) - c_p(T_d - T_a)}{c_p(T_c - T_b)} = 1 - \frac{T_d - T_a}{T_c - T_b}$$

$$= 1 - \frac{T_a}{T_b}, \quad \text{or} \quad 1 - \left(\frac{p_b}{p_a}\right)^{\frac{1-k}{k}} = 1 - \left(\frac{p_a}{p_b}\right)^{\frac{k-1}{k}}$$

$$\eta_{\text{Carnot}} = \frac{T_b(\phi_2 - \phi_1) - T_a(\phi_2 - \phi_1)}{T_b(\phi_2 - \phi_1)}$$

$$= 1 - \frac{T_a}{T_b},$$

that is,

$$\eta_{\text{Otto}} = \eta_{\text{Brayton}} = \eta_{\text{Carnot}}.$$

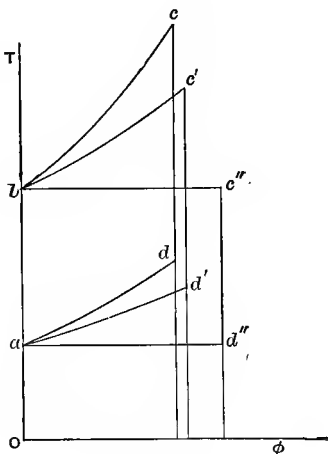


FIG. 69.

Such a case might arise when all three engines were using the same combustible, thus necessitating that the initial pressure be limited to prevent pre-ignition. Since the efficiency of all three cycles will be the same, a decision as to the most advantageous to use must be based upon other considerations.

The differences between the three cycles are best shown in tabular form:

	Pressure Range.	Volume Range.	Temp. Range.
Brayton.....	Minimum	Intermediate	Intermediate
Otto.....	Maximum	Minimum	Maximum
Carnot.....	Intermediate	Maximum	Minimum

From the table the Carnot is shown to be the least desirable in that it requires the largest engine, although the fluctuations in its rotative effect are not as great as in the Otto, so that the working parts would not need to be as heavy as in the latter. But besides this the Carnot cycle has to be discarded as unfeasible because although the Diesel motor approximates it on the forward stroke no gas-engine has as yet been invented to give isothermal compression.

The Brayton cycle has the minimum change of pressure and therefore the least fluctuating rotative effect. Its volume range is, however, greater than that of the Otto. But again the maximum temperature reached in the Brayton is less than in the Otto, so that less jacket cooling of the cylinder will be necessary and the interchange of heat between gas-metal-gas will also be less.

By keeping the gas and air separate, as in the Diesel motor, it is possible to utilize much higher initial compression, and then the only limit theoretically is the resistance of the machine to pressure or temperature or both. We will next compare the Brayton and Otto cycles with reference to maximum pressure and maximum temperature.

Let  $bcc'$  (Fig. 70) represent the maximum safe pressure,

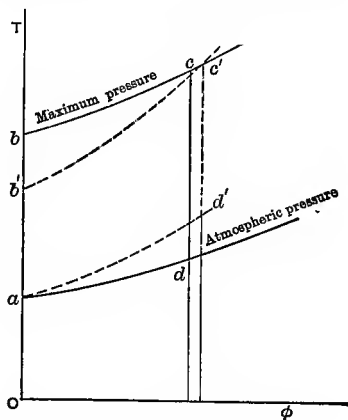


FIG. 70.

whether it be the sustained pressure of the Brayton cycle or the momentary maximum of the Otto. Further, let  $ad$  be atmospheric pressure, the lowest pressure of exhaust.

In the Brayton cycle initial compression is carried up to  $b$ , while in the Otto cycle it must stop at some intermediate point  $b'$ , such that the further increase

due to heating will carry it just to  $c'$ . As the charges are identical the area under  $bc$  equals the area under  $b'c'$ . But the heat rejected in the Otto exceeds that in the Brayton. Whence it follows that given a Brayton and an Otto engine working between the same limits of pressure the former is the more efficient and possesses the smaller temperature range.

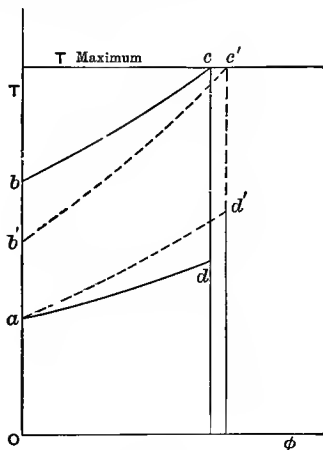


FIG. 71.

Similarly let  $cc'$  (Fig. 71) represent the safe maximum temperature. Then  $abcd$  and  $ab'c'd'$  will represent the Brayton and Otto cycles, respectively, working between atmospheric pressure and temperature and maximum temperature  $cc'$ . Again the Brayton cycle shows the greater efficiency, although both possess the same temperature range, and the Otto cycle has a trifle the smaller pressure range.

**Deduction from the General Theory of Gas Cycles.**—On pages 161–163 we discussed the general case of a gas cycle consisting of two pairs of polytropic curves, and

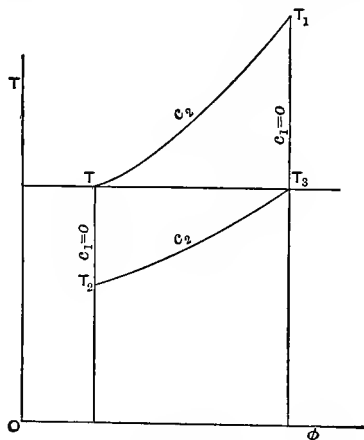


FIG. 72.

obtained as the condition of maximum work per cycle that

$$T = \sqrt{T_1 T_2} = T_3.$$

This gave for maximum work and the corresponding efficiency the expressions,

$$W_{\max.} = \frac{c_2 - c_1}{A} (\sqrt{T_1} - \sqrt{T_2})^2$$

and

$$\eta = \frac{(c_2 - c_1)(\sqrt{T_1} - \sqrt{T_2})}{c_1 \sqrt{T_2} + c_2 \sqrt{T_1}}.$$

*Second Special Case.*—If  $c_1 = 0$  the cycle consists of two isentropic and two polytropic curves, as shown in



Fig. 72. The maximum work per cycle between given temperature limits reduces therefore to

$$W_{\max.} = \frac{c_2}{A} (\sqrt{T_1} - \sqrt{T_2})^2,$$

and the corresponding thermal efficiency to

$$\eta_{\max. \text{ work}} = \frac{\sqrt{T_1} - \sqrt{T_2}}{\sqrt{T_1}} = \frac{\sqrt{T_1 T_2} - T_2}{\sqrt{T_1 T_2}} = \frac{T - T_2}{T},$$

whence as on p. 176,

$$= 1 - \frac{T_2}{T} = 1 - \left(\frac{v}{v_2}\right)^{k-1}.$$

The dimensions of the engine to give the maximum amount of work per cycle between any given temperature limits may be found as follows:

The equation of the  $Tv$ -projection of an adiabatic gives

$$T_2 v_2^{k-1} = T v^{k-1},$$

whence

$$\begin{aligned} \left(\frac{v}{v_2}\right)^{k-1} &= \left(\frac{\text{Clearance}}{\text{Clearance} + \text{piston displacement}}\right)^{k-1} \\ &= \frac{T_2}{T} = \frac{T_2}{\sqrt{T_1 T_2}} = \sqrt{\frac{T_2}{T_1}}, \end{aligned}$$

or

$$\text{Clearance} = \frac{\text{P.D.} \left(\frac{T_2}{T_1}\right)^{\frac{1}{2(k-1)}}}{1 - \left(\frac{T_2}{T_1}\right)^{\frac{1}{(2k-1)}}}.$$

**Variation of Efficiency with Load.**—The efficiency of the internal-combustion engine varies not only with the type of cycle utilized, but also with the form of regulation adopted in any particular engine.

A. Engines working upon the Otto cycle are governed by the following methods:

- I. The omission of an occasional charge, known as hit-or-miss regulation.
- II. Throttling of the fuel supply, known as quality regulation.
- III. Variation of the amount of charge, obtained by throttling the mixture or by a variable cut-off, known as quantity regulation.
- IV. Regulation by varying the time of ignition.
- V. Various compound methods involving two or more of the above.

The effects of these various methods upon the efficiency of the engine under variable load can be clearly illustrated with the ideal cycle in the  $T\phi$  plane.

I. *Hit-or-miss Regulation.*—Whenever a charge of fuel is omitted the scavenging action of the air combined with the constant action of the jackets serves to cool the cylinder walls and the admission ports to a temperature below the average, so that the next charge will be cooler and of greater density.

Therefore the first explosion immediately following a “miss” stroke will be more powerful than the average as it contains a greater quantity of fuel. Thus if  $abc'd'$ ,

Fig. 73, represent the average cycle, the cycle following the "miss" will be represented by some diagram such as  $abc''d''$ . In case of light load it may happen that two or more miss strokes occur successively, in which case the walls may be cooled to such an extent that the resulting low temperature of the next charge retards its combustion. This is equivalent to exhausting part

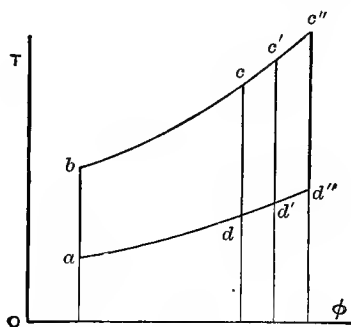


FIG. 73.

of the charge unburned and results in a smaller heat diagram  $abcd$ .

It will be noticed that with constant port openings and constant speed the throttling action during admission is always the same, so that the pressure at the end of admission is always practically the same. Hence if the minor variations in percentage mixture caused by the varying density and composition of the clearance gases may be overlooked, the effect of a miss stroke is simply to vary the power of the succeeding stroke but not its efficiency.

Variation in load is met automatically by the engine by varying the frequency of the miss strokes. Therefore this type of regulation must give practically the same thermal efficiency for all loads, but it has the mechanical disadvantage of requiring more massive construction and heavier fly-wheels if good regulation in speed is to be obtained. In practice engines governed by hit-or-miss regulation usually show a somewhat greater fuel economy than is obtained by the other methods.

II. *Quality Regulation.*—A full charge of fuel and air is taken each suction stroke, but the percentage mixture is varied by the governor which controls a throttle valve placed in the fuel supply pipe. The initial pressure being the same the efficiency should apparently be the same, but experiments made upon explosive mixtures\* of fuel and air have shown that the rate of combustion is retarded by dilution, and this dilution may readily be so great as to prevent combustion or at least to delay it until after the charge has left the cylinder. This unburned or wasted heat, combined with the greater relative effect of the cylinder walls upon a smaller heat charge (see next chapter), serves to diminish the efficiency as the load falls off.

Thus if  $abcd$ , Fig. 74, represent the heat available at full load,  $abc'd'$  would represent the heat available

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\* See report by Author in *Technology Quarterly*, Sept., 1900, pp. 248-259.

for some smaller load if it were not for the above mentioned losses. Due to dilution, however, a certain portion of this theoretically available heat, say  $c''c'd'd''$ , either is lost through imperfect combustion or is so much delayed as to be generated after the charge is

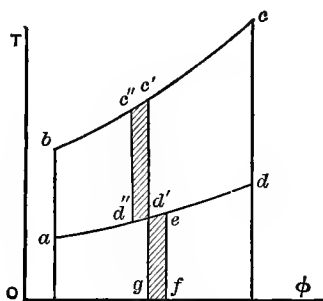


FIG. 74.

exhausted and must therefore be added to the exhaust heat, as  $d'efg$ .

III. *Quantity Governing*.—In this system of regulation the percentage mixture is adjusted by hand to give the maximum explosive value and is then maintained constant, but the quantity of the mixture admitted to the cylinder is controlled by a valve placed in the pipe leading from the mixing chamber to the cylinder. The action of the governor being either to throttle the valve throughout the suction stroke or to close it completely at varying percents of the stroke so as to adjust the quantity of charge to the load.

*a. Throttling.*—Regulation by throttling involves a drop of pressure in the throttle valve so that the pressure of admission is less than the pressure of discharge. This produces a negative loop in the indicator card

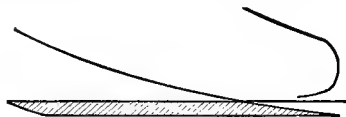


FIG. 75.

whose area measures the work required to overcome the friction losses during admission and exhaust. See Fig. 75.

*b. Automatic Cut-off.*—Here the valve is kept wide open until cut-off occurs. Hence the charge is admitted

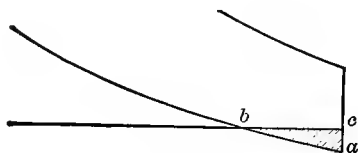


FIG. 76.

at atmospheric pressure up to cut-off, and from that point expands adiabatically to the end of the stroke. On the compression stroke this adiabatic operation is traversed in the reverse direction until atmospheric pressure is reached. The friction loss of the preceding method is therefore avoided. Figs. 76 and 77.

The weight of the burnt gases is practically always the same, as they are discharged under the same pressure,

while the weight of the new charge varies with the amount of throttling and with the time of cut-off, so that the proportion between the burnt gases and the new charge is a variable, that is, the burnt gases serve to increase the dilution of the gaseous mixture as the load decreases. Quantity regulation thus possesses not

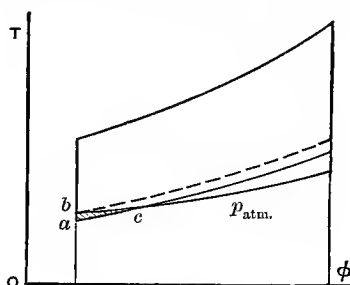


FIG. 77.

only the losses due to its throttling action but also to a small extent those due to quality regulation.

IV. *Regulation by Varying the Time of Ignition.*—Ignition must occur before the end of the compression stroke so that by the time compression is finished the flame shall have had opportunity to spread throughout the entire charge and produce maximum pressure. For any given load the spark should therefore be adjusted by hand so as to give best results. In quality regulation and to a lesser extent in quantity regulation the rate of flame propagation falls off as the load decreases, so that if best results are to be attained the spark must be correspondingly advanced.

Advantage is taken of this in many smaller motors, as for example in automobiles and motor boats, to adjust the power of the engine to any desired condition by shifting the spark by hand. Thus practically the same quantity of fuel is admitted each stroke and more and more of it wasted as the load falls off.

Assuming for the moment that the explosion is absolutely instantaneous the effect of retarding the

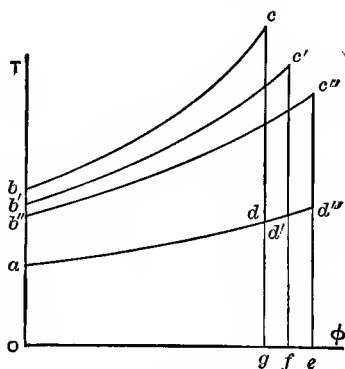


FIG. 78.

spark is clearly shown in Fig. 78. Let  $abcd$  represent normal running under full load. If the load falls off somewhat the spark is retarded until the charge has expanded part way down the compression line  $ba$ , say to  $b'$ ; it then burns at constant volume  $b'c'$  generating the same amount of heat as at  $bc$  but with lower temperatures and hence greater entropy increase. The result is therefore an increase in the exhaust heat



by an amount  $dd'fg$ . The later the spark the greater the increased exhaust. In other words the effect is similar to that which would be obtained by an increasing clearance space.

*B.* Engines working upon the Diesel cycle are governed by varying the time interval during which fuel is admitted to the cylinder.

Let it be assumed that there is always sufficient oxygen present to insure complete combustion of the

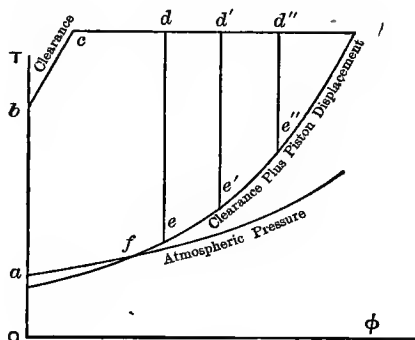


FIG. 79.

fuel at maximum load. If the fuel is injected so as to supply heat isothermally the shape of the cycle with increasing load is shown by  $abcdef$ ,  $abcd'e'f$ , etc., Fig. 79, where the areas under  $bcd$ ,  $bcd'e'$ , etc., represent the heat generated by the fuel and  $de$ ,  $d'e'$ , etc., represent the adiabatic expansion from the moment the fuel supply is discontinued to the end of the stroke. It is evident that the drop of temperature  $de$ ,  $d'e'$ ,

etc., experienced by the additional fuel decreases so that the average efficiency falls off as the load increases. If, for example, injection of fuel could be continued to the end of the stroke, the ideal efficiency would be but slightly greater than one half that of the Carnot efficiency although at ordinary load it is nearly equal to it.

**Comparison of the Different Gas-cycles.** — Because of its successful application in commercial use it is necessary to deduce an analytical expression for the efficiency of the Diesel cycle. Certain assumptions must be made at the start in order to give this cycle that simplicity of form which will make the analytical expression useable. On actual indicator cards taken from Diesel engines the process most variable in character is that of the admission of the fuel. This fluctuates more or less in magnitude upon the different cards, but in general may be fairly well represented by a constant pressure line. Such an assumption is justifiable from a theoretical point of view, as the Diesel cycle attempts to utilize the great increase in efficiency which comes from high initial compression. In order that this gain may be a maximum, such compression should be carried to the upper pressure limit, so that when the fuel is injected no further increase in pressure is permissible and any decrease in pressure would diminish the temperature range of the heat. This differs from the original Diesel cycle which assumed

isothermal combustion, and was based upon the mistaken idea that isothermal generation of heat was essential to the attainment of maximum efficiency. Assuming, then, that the ideal Diesel cycle may be represented by constant pressure reception of heat and constant volume rejection of heat, these two processes, being connected by adiabatic expansion and compression, it follows that the expression of thermal efficiency is equal to  $C_p$  times the increase in temperature during the injection of the fuel, minus  $C_v$  times the decrease in temperature during release, divided by the heat received. A fundamental distinction between this expression and the common expressions for the Otto, Joule, and Carnot cycles is to be noted in the retention of the specific heats. Thus in the Otto cycle  $C_v$  appears in all three terms, and similarly in the Joule cycle  $C_p$  appears in all three terms, so that this common factor cancels in all these expressions making the efficiency absolutely independent of the character of the working substance, whereas in the expression for the efficiency of the Diesel cycle the ratio of these specific heats remains, so that the efficiency is dependent to a certain extent upon the character of the working fluid used. By means of thermodynamic relations easily established between the temperatures and the volumes of the corners of the cycle it is possible to eliminate from the expression of the efficiency all the temperatures and to retain in their places certain ratios

of volumes. The expression may be given in the following final form:

$$\text{Eff.} = 1 - \frac{\left( \frac{\text{Vol. at cut-off}}{\text{Vol. of clearance}} \right)^k - 1}{k \left( \frac{\text{Vol. at cut-off}}{\text{Vol. of clearance}} - 1 \right)} \left( \frac{\text{Vol. of clearance}}{\text{Vol. of cl. + piston dis.}} \right)^{k-1},$$

or

$$\text{Eff.} = 1 - \frac{\left( \frac{\text{C.O.}}{\text{Cl.}} \right)^k - 1}{k \left( \frac{\text{C.O.}}{\text{Cl.}} - 1 \right)} \cdot \left( \frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}} \right)^{k-1}$$

This expression is of the same form as that of the efficiency of the Otto, Joule, and Diesel cycles, except that a certain coefficient different from unity is introduced as a multiplier in the last term. Thus, as we have seen, the expression for the Otto efficiency takes the form:

$$\text{Eff.} = 1 - \left( \frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}} \right)^{k-1}$$

An investigation of this coefficient:

$$\frac{\left( \frac{\text{C.O.}}{\text{Cl.}} \right)^k - 1}{k \left( \frac{\text{C.O.}}{\text{Cl.}} - 1 \right)}$$

shows us that as the cut-off increases the numerator increases more rapidly than the denominator, since the expression:

$$\left( \frac{\text{C.O.}}{\text{Cl.}} \right)^k,$$

as  $k$  is greater than one, increases more rapidly than the simple ratio

$$\left( \frac{\text{C.O.}}{\text{Cl}} \right).$$

Therefore as the load increases the value of this coefficient increases so that the efficiency of the cycle diminishes. The maximum values of the efficiency of the Diesel cycle are therefore obtained with the smaller loads, and as the load continues to decrease this value increases and approaches as a limit that of the Otto cycle which theoretically will be attained at zero load.

It is evident, therefore, that with the same compression the Diesel cycle is not as efficient as either the Otto, Joule, or Carnot cycles, but that its great practical efficiency has been attained by making use of the increase in efficiency in any of these cycles which may be produced by utilizing smaller clearance space. This, as we have seen, is accomplished by keeping the fuel separate from the air during the compression and injecting the fuel whenever the desired pressure has been attained. As soon as this method of separate compression is introduced there is no longer any necessity of utilizing such low compressions in the Otto and Joule cycles, so that it would be possible to have in these cycles exactly the same compression as that realized in the Diesel engine, and in such a case this cycle would be inferior to either of the others. It is probable that, as advantage is being taken of this

high compression, the initial compression would be carried to the ultimate upper limit consistent with safety, so that no further increase in pressure would be permitted during the burning of the fuel. In such a case it would no longer be possible to utilize the Otto cycle, and one would be forced back upon the Joule cycle.

In case compression is not carried to the maximum upper limit and a still further increase is permissible in the cylinder during combustion, it would then be possible to utilize burning at constant volume and therefore receive the heat in the manner adopted in the Otto cycle. If in connection with this constant volume reception of heat there could at the same time be combined a constant pressure rejection of heat, an efficiency could be obtained greater than that of either the Otto or the Joule cycle. A cycle thus constituted would evidently be but the old Atkinson cycle. Proceeding in a manner similar to that adopted in the case of the Diesel cycle, an expression for the efficiency of the Atkinson cycle can be deduced in terms of  $k$  and certain volume ratios. This expression may be given the final form:

$$\text{Eff.} = 1 - \frac{k \left( \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} - 1 \right)}{\left( \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} \right)^k - 1} \cdot \left( \frac{\text{Vol. of clearance}}{\text{Vol. after admission}} \right)^{k-1}$$

This expression is seen to have the same form as that

of the Otto cycle with the exception that the coefficient of the second term is other than unity. An inspection of this coefficient:

$$\frac{k \left( \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} - 1 \right)}{\left( \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} \right)^k - 1}$$

shows that the efficiency is dependent upon the value of  $k$  for the given substance and also dependent upon the volume after expansion; this volume necessarily increasing as the quantity of charge increases.

As the volume after suction may be considered a fixed quantity, it is evident that as the volume after expansion increases the ratio of these two volumes increases at a slower rate than the same ratio raised to the  $k$  power, so that with increasing load the denominator of this coefficient increases at a higher rate than the numerator, so that the coefficient as a whole decreases in value as the load increases, a result which is exactly the opposite of the history of the corresponding coefficient for the efficiency of the Diesel cycle.

An inspection of these two cycles in the temperature entropy plane makes clear this fundamental difference in these two cycles. Start with the same initial compression in both cycles. In the Diesel cycle, on the one hand, the constant pressure curve drawn through the end of compression and the constant volume curve

drawn through the beginning of compression approach indefinitely as the load increases, so that whereas at the beginning or at a very light load the efficiency is equal to that of a Carnot cycle, we find that the successive quantities of heat taken in as the load increases suffer a smaller drop in temperature than the earlier quantities, and therefore are utilized less efficiently, so that the net result of an increasing load is to bring down the average efficiency at which the heat is utilized. In the Atkinson cycle, on the other hand, since a constant volume curve drawn through the end of compression and a constant pressure curve drawn through the beginning of compression diverge as the load increases, the range of temperature experienced by successive quantities of heat increases so that the efficiencies of the final portions are greater than the efficiencies of the earlier portions, and therefore the result of an increasing load is to increase the average efficiency at which the heat is utilized. As the load falls off this coefficient increases in value and approaches unity as a limit, this value, however, being attained only at zero load, when the efficiency of the Atkinson cycle will be reduced to that of the Otto cycle.

This discussion makes it evident that the Carnot cycle is not the most efficient cycle when adapted to gas engine work. The requirement that all the heat should be received at the highest possible temperature, and that all the heat which must be rejected should



be rejected at the lowest possible temperature in order to obtain the maximum amount of work from a given quantity of heat, is necessarily fundamentally true. If the source of heat were at a definite upper temperature, the fulfillment of this fundamental requirement would necessarily require the use of the Carnot cycle, but if the heat is generated by combustion in a confined space the very act of combustion will result in an increase in the temperature of the substance in this confined space, so that any further combustion will occur at a higher temperature. In other words, successive quantities of heat will be generated in the confined space at successively higher temperatures. In fact, to prevent the temperature from rising in order to obtain the requirement of the Carnot cycle, would necessitate careful adjustment between the rate of fuel consumption and the rate of work production by the piston.

In the Diesel cycle the heat is generated at such a rate that the pressure remains practically constant. In the Otto cycle the generation of heat is so nearly instantaneous that the process occurs practically at constant volume. In both of these cases the temperature rises continuously throughout the period of combustion, and, other things being equal, the heat received along the constant pressure curve is capable of better utilization than that received along the isothermal curve, and similarly the heat received at constant volume is capable of better utilization than that received

at constant pressure. This is very clearly illustrated by the relative slopes of the three curves in the temperature entropy plane; as the steeper the curve the smaller the increase in entropy, and therefore the smaller the amount of heat which must eventually be rejected. This analysis shows, therefore, that for the same initial compression the constant volume curve is the one which has the highest theoretical possibilities. It might seem at first sight that a curve even steeper than the constant volume could be utilized. A little thought, however, will show that such a process could only be taking place while the piston was on its return stroke. In such a case, therefore, it would be better to wait until this return stroke were completely finished, so that all of the heat might be generated at the higher temperatures consequent upon such an increased compression. As we have seen, a cycle consisting of constant volume production of heat, a constant pressure rejection of heat, combined with adiabatic compression and expansion, gives a higher efficiency than the straight Otto cycle. Such an Atkinson cycle, however, although it fulfills the requirements for the reception of heat at maximum temperature, does not at the same time fulfill the requirements for the rejection of heat at minimum temperature. It is true that the Atkinson cycle expanding down to initial pressure probably rejects the heat at as low a temperature as is practicable, but there nevertheless

exists the theoretical possibility of continuing this expansion down to the initial temperature and then closing the cycle by means of an isothermal compression. Possibly such an operation could be realized to a certain extent by the injection of cold water into the cylinder during this compression stroke. It is interesting at least to compare the expression for the thermal efficiency of this cycle with those already considered.

The method of obtaining this expression is of the same general character as that adopted for the previous cycles; namely, to draw the cycle in the temperature entropy plane and then to determine the heat quantities as represented by the areas under the proper lines. The only new feature in this particular formula is the heat exhausted during the isothermal compression. This may readily be obtained by multiplying the temperature of exhaust by the increase in entropy, which may be determined during the constant volume generation of heat. Going through the steps involved in such work and eliminating the temperatures from the formula, as in the preceding cases, we arrive eventually at the following expression:

$$\text{Eff.} = 1 - \frac{(k-1) \log_e \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} \left( \frac{\text{Cl. vol.}}{\text{Vol. after ad.}} \right)^{k-1}}{\left( \frac{\text{Vol. after expansion}}{\text{Vol. after admission}} \right)^{k-1} - 1}$$

This expression takes a general form similar to those

of the preceding cycles, the only difference being that this coefficient is smaller than any of the preceding for the same compression and the same amount of fuel. We further see that the denominator increases more rapidly than the numerator as the volume after expansion increases, because this enters the denominator as a factor proportional to its  $k-1$  power, while in the numerator it appears simply as a logarithm.

An instructive tabulation of the results obtained from these different cycles may be made by writing them in the general form

1 - Eff. = the fractional part of the energy wasted.

$$\text{Diesel, } 1 - E_D = \frac{\left(\frac{\text{C.O.}}{\text{Cl.}}\right)^k - 1}{k\left(\frac{\text{C.O.}}{\text{Cl.}} - 1\right)} \times \left(\frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}}\right)^{k-1};$$

$$\text{Otto, } 1 - E_O = 1 \times \left(\frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}}\right)^{k-1};$$

$$\text{Atkinson, } 1 - E_A = k \frac{\left(\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}} - 1\right)}{\left(\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}\right)^k - 1} \left(\frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}}\right)^{k-1};$$

$$\text{Max. } 1 - E_{\text{max}} = \frac{(k-1) \log_e \frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}}{\left(\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}\right)^{k-1} - 1} \left(\frac{\text{Cl.}}{\text{Cl.} + \text{P.D.}}\right)^{k-1}$$

If the Otto cycle be taken as the standard of reference, the waste heat of the other cycles may be expressed in terms of the waste heat of the Otto by means of the ratios

$$\frac{\text{Diesel}}{\text{Otto}} = \frac{\left(\frac{\text{C.O.}}{\text{Cl.}}\right)^k - 1}{k\left(\frac{\text{C.O.}}{\text{Cl.}} - 1\right)};$$

$$\frac{\text{Atkinson}}{\text{Otto}} = k \frac{\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}} - 1}{\left(\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}\right)^k - 1};$$

$$\frac{\text{Maximum}}{\text{Otto}} = \frac{(k-1) \log_e \frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}}{\left(\frac{\text{Cl.} + \text{P.D.}_{\text{ex}}}{\text{Cl.} + \text{P.D.}_{\text{ad}}}\right)^{k-1} - 1}.$$

## CHAPTER X.

### THE GAS-ENGINE INDICATOR CARD.

**Physical Conditions of the Problem.**—The direct application of the  $T\phi$ -analysis to an actual gas-engine, although simple in theory, is difficult in practice. To find the various heat losses from the study of an indicator card necessitates an exact knowledge of the amount of heat available per revolution. In a steam-engine this may be determined with a fair degree of accuracy, provided the boiler and exhaust pressures remain constant and the cut-off varies within narrow limits, as then each pound of steam contains a known quantity of heat and the weight required to develop a given power is measurable. In a gas-engine the operation is more involved. The energy is not drawn from a reservoir but is generated each working stroke in the cylinder. Of course the gas can be metered and the average consumption found per working stroke, but how is the corresponding average indicator card to be determined? It is easy to obtain cards from the same engine showing quick, moderately fast, and slow burning of the fuel. (See Figs. 81 and 82.) How are the

corresponding percentage mixtures and the absolute quantity of fuel to be found for each different card? In the hit-or-miss type of governing the "miss" stroke, acting as a scavenger, cools the cylinder so that the following charge is heated less during admission, is expanded less, and hence a greater weight enters the cylinder than would directly after an explosion. The result is the percentage mixture of gas, air, and burnt products varies throughout the explosions for a complete firing cycle. Perhaps the best that can be done is to take each time the complete set of cards for the firing cycle and draw an equivalent average card. If governing is effected by throttling the fuel gas there seems to be no method of determining the relative proportions of gas and air; if the governor controls either by throttling or by cutting off the admission of a mixture of known composition, the relative proportions between it and the burnt products will still be unknown. Possibly in engines of the Körting type, where the gas and air are drawn separately into cylinders of known capacity, it may be possible to attain fairly accurate knowledge of these quantities per stroke.

The best that can usually be done is to keep the load as constant as possible and use average cards and average fuel consumption. In any case the gas and air must be measured independently and this in the case of a large gas-engine will require a large gas-meter, or some form of displacement tanks for measuring the

air. If nothing else is at hand, an anemometer can be used.

Having determined the average indicator card and the average heat applied per explosion, the next difficulty is the determination of the specific heat at constant volume and the ratio  $\frac{c_p}{c_v}$ . The difficulty is here two-fold: first, the character of the mixture is doubtful, due to the varying composition of the burnt gases in the clearance space; secondly, our knowledge of the variations of the specific heats at high temperatures is very incomplete. Thus MM. Mallard and Le Chatelier found by experiment the values:

Carbonic dioxide . . . .	$c_v = 0.1477 + 0.000176t$
Water vapor . . . . .	$= 0.3211 + 0.000219t$
Nitrogen . . . . .	$= 0.170 + 0.0000872t$
Oxygen . . . . .	$= 0.1488 + 0.0000763t$

If the weight of each component present in the charge is known the value of  $c_v$  for the mixture is given by

$$c_v = \frac{\sum wc_1}{\sum w} + \frac{\sum wc_2}{\sum w} \cdot t.$$

Due to the chemical changes during combustion the value of  $c_v$  differs during different parts of the cycle.

The value of  $k$  is likewise a variable. Clerk\* finds that 1.37 and 1.28 are better values for compression

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\* *The Engineer*, March 1 and 8, 1907, pp. 205 and 236.



and expansion respectively than the customary value 1.4.

The specific volume of the charge is decreased by combustion due to the rearrangement of the molecules into new combinations. Thus for some samples of Boston illuminating-gas the ratio of final to initial specific volumes was for different percentage mixtures of gas and air as follows:

Gas to air . . . . .	1:6	1:7	1:8	1:9
$\frac{\text{Final volume}}{\text{Initial volume}}$ . .	0.964	0.969	0.972	0.975

Clerk \* states that 1 volume of gas and 5 of air shrink 4 per cent. and 1 volume of gas and 7 of air shrink 3 per cent. and 1 volume of gas and 10 of air shrink 2.2 per cent.

To transfer the gas-engine indicator card to the  $T\phi$ -plane requires therefore

- (1) Average percentage mixture of the charge and the corresponding average card.
- (2) The laws of variation of  $c_v$  and  $k$ .
- (3) The degree of combustion so that the shrinkage in specific volume may be calculated.

If all these conditions are fulfilled the corresponding temperatures and entropies can be calculated and the desired plot obtained.

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\* Min. Proc. Inst. Civ. Eng., vol. CLXII, p. 314.

If the temperature can be obtained independently by direct measurement for any point of the cycle it serves as a check upon the other observations. This would require some thermo-electric device where the circuit is closed at the proper moment by the engine. The difficulties of such a method are considerable, and even when accurately obtained the indicated temperatures are not the desired average temperature but the temperature of the charge at the point at which the thermo-electric couple is introduced. Various rough approximations have been given for the temperature of the charge at the end of admission.

Boulvin, in the *Entropy Diagram*, assumes it to be a mean between that of the surrounding air and of the jacket water. He further neglects the chemical change due to combustion and assumes that the values of  $c_p$ ,  $c_v$ , and  $k$  of the charge hold sufficiently well for the burnt products, and further assumes that  $c_p$  and  $c_v$  remain constant at high temperatures.

Prof. Burstall, in the *Second Report of the Gas-engine Research Committee* (Proc. Inst. Mech. Eng., 1901, p. 1083), takes the temperature as generally not differing greatly from the jacket temperature. He finds the computed temperature, however, to be considerably higher than this in all cases, the maximum and minimum differences being about  $90^\circ$  and  $32^\circ$  F. He also uses the variable values of  $c_v$  given on page 175.

Prof. Reeve, in *The Thermodynamics of Heat-engines*,

states that he usually assumes the initial temperature at the convenient round number of  $600^{\circ}$  absolute Fahrenheit.

**Ideal Indicator Cards.**—Having settled these preliminaries, i.e., having determined the average card, the average heat supplied per cycle, the values of  $c_v$  and  $k$  for the mixture, the shrinkage, and the initial temperature, the next step is to draw the ideal card.

Thus if  $T_0$  = estimated or measured temperature at admission,

$p_0$  = initial pressure observed from card,

$v_0$  = volume of clearance plus piston displacement,

$v_1$  = volume of clearance,

$Q_1$  = B.T.U. received per cycle,

$Q_2$  = B.T.U. rejected per cycle,

then, referring to Fig. 73, the following equations give the proper values of the coordinates of the points 1, 2, and 3.

$$T_1 = T_0 \left( \frac{v_0}{v_1} \right)^{k-1}, \quad k \text{ may usually be assumed as } 1.38;$$

$$T_2 = \frac{Q_1}{c_v} + T_1;$$

$$T_3 = \frac{T_0 \cdot T_2}{T_1}; \quad (\text{See p. 161.})$$

$$Q_2 = \frac{T_0}{T_1} \cdot Q_1;$$

$$\phi_2 = \phi_3 = c_v \log_e \frac{T_2}{T_1} + \phi_0.$$

The ideal cycle will thus be different for each test as in each case the gaseous mixture is different, so that in each case the special values of  $c_v$  and  $k$  must be determined.

This is, as we have seen, a difficult problem, but, fortunately, the variations in  $c_v$  and  $k$  are not great between different tests on the same engine and between the gaseous mixtures arising from different fuels, so that these variations may be overlooked without introducing too great an error in the results.

Thus Prof. Lucke, in his *Gas-engine Design*, establishes a standard reference diagram by considering the working fluid to be air filling the cylinder at the end of the admission stroke under 14.7 lbs. pressure and at 32° F. The air is supposed to pass through the usual Otto cycle, during ignition receiving at constant volume all the heat generated per cycle in the actual engine. Such a device, although recommended by its simplicity and admirably adapted to the end in view, viz., the establishment of diagram factors to assist in the design of engines of any required power and working with any desired fuel, is unsuitable for the purposes of heat analysis.

In the opinion of the Committee of the Institution of Civil Engineers (see Min. Proc., 1905, pp. 324 and 326), "it would introduce some uncertainty and difficulty, without adequate compensating advantage, to make the standard engine-cycle depend on a knowledge of

the physical constants of the exact mixture used. The discussion of the constant for various mixtures of gases already given shows that, apart from the unknown change at high temperatures, these constants do not differ by more than 2 per cent. to 5 per cent. from those of air in such mixtures as are used in gas-engines. The advantages of simplicity and definiteness in the standard are so great that the Committee recommend that the standard engine should be taken to work with a perfect gas of the same density as air. This in no way prevents any one from discussing the distribution of heat losses in any particular trial with constants adjusted to any particular mixture of gases. But it does render more definite the statement of the relative efficiency (cylinder efficiency), without, in the opinion of the Committee, introducing any error of practical importance.

“The standard engine of comparison is therefore a perfect air-gas engine operated between the same maximum and minimum volumes as the actual engine, receiving the same total amount of heat per cycle, but without jacket or radiation loss, and starting from one atmosphere and the selected initial temperature of 139° F. Its efficiency is the same as that of a Carnot engine working through the temperature range  $T_1 - T_0$  or  $T_2 - T_3$  (Fig. 80). The efficiency is given by a very simple expression, depending only on the dimensions of the cylinder and independent of the heat supply or the maximum temperature. If the heat supply is

increased by using a richer charge, the work done is increased, but not the efficiency. The pressure-volume or temperature-entropy diagram of the standard cycle can be drawn on the assumptions just stated, which, in the opinion of the Committee, do not essentially differ from those in actual engines."

The only effect of using the "standard air cycle" instead of the actual ideal cycle will be in each case to change the values of the ideal and cylinder efficiencies a few per cent. Thus the ideal efficiency varies as shown in the following table:

(Clearance + piston displacement) ÷ clearance	$\eta = 1 - \left( \frac{\text{clearance}}{\text{Clearance} + \text{piston displacement}} \right)^{k-1}$	
	$k = 1.40$	$k = 1.37$
2	0.246	0.225
3	0.36	0.332
4	0.43	0.399
5	0.47	0.45
7	0.55	0.51
10	0.61	0.57
20	0.70	0.67
100	0.85	0.82

In ordinary commercial gas-engines of moderate size, the cylinder efficiencies, referred to the air-engine standard, vary from 0.5 to 0.6. A comparison of the efficiencies of actual engines and the air-engine standard is given in Mr. Dugald Clerk's paper, "Recent Developments in Gas-engines," in *Min. Proc. Inst. C. E.*, Vol. CXXIV, p. 96.

In discussing the effect of size, speed, clearance, time of firing, etc., upon engines using the same fuel, the air-standard cycle can be used to good advantage, but in studying the heat losses due to different fuels, etc., an error is liable to be introduced unless the proper value of  $k$  is used for each mixture.

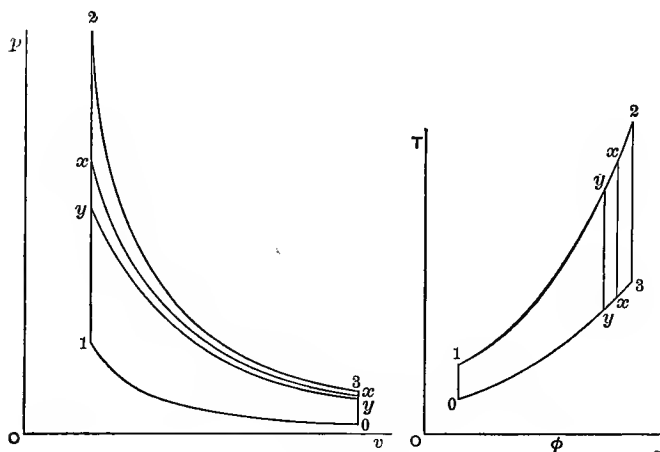


FIG. 80.

**Cylinder Efficiency.**—The discrepancy between the ideal and the actual cards is due to three influences:

First, the shrinkage due to combustion would cause the expansion line of the ideal card to occur at pressures reduced proportionally to its magnitude. Less work is thus done during the expansive stroke while the work of compression is undiminished, thus reducing the net work obtainable from the cycle. This new

expansion line would occupy some such position as  $xx$ . That portion of the ideal card beyond  $xx$  is, therefore, from the nature of the substance unattainable. Fig. 80.

Next, there is the loss due to incomplete combustion. This may be caused by a mixture of such proportions that the fuel cannot all burn, or by one in which the combustion is retarded so much as to be unfinished at exhaust; in either case the full heat value of the charge is not developed in the cylinder. Although such a charge does not burn at constant volume, but is burning throughout part or all of the expansive stroke, and cannot thus be represented on the ideal cycle, yet the magnitude of the lost heat may be represented by drawing  $yy$  so that the area between  $xx$  and  $yy$  equals this loss. The area under  $1y$  is thus equal to the heat actually generated in the cycle.

Finally, there is the loss due to heat interchange between the gas and the metal which produces the discrepancy between that portion of the ideal card to the left of  $yy$  and the actual card.

It is almost never possible to locate  $xx$  and  $yy$ , as the amount of shrinkage in the cylinder has to be estimated from the analysis of the burnt gases in the exhaust-pipe. But with delayed combustion the gas may not be the same in the two places. The best that can be done is to draw  $xx$  to correspond to the shrinkage for complete combustion.

The losses due to incomplete combustion and to heat



interchanges with the cylinder walls are thus combined, due to our inability to locate  $yy$ . But by suitably regulating the percentage mixture it is possible nearly to eliminate the loss due to incomplete combustion, although as the governor controls the percentage mixture either of air and gas, or of mixture and burnt products, slow-burning charges sometimes result.

The cylinder efficiency, i.e., the ratio of the actual to the ideal card, is thus limited by two different factors, the physical properties of the fuel—shrinkage and rate of flame propagation—and the influence of the cylinder walls. It is in tracing out the heat interchanges between the charge and the cylinder walls that the  $T\phi$ -analysis is of greatest value.

**Actual Indicator Cards.**—To plot the  $T\phi$ -projection of a gas-engine indicator card the temperature and entropy for a sufficient number of points can be calculated from the equations

$$T_x = \frac{T_0}{p_0 v_0} \cdot p_x v_x = c \cdot p_x v_x,$$

$$\phi_x - \phi_0 = c_v \log_e \frac{p_x}{p_0} + c_p \log_e \frac{v_x}{v_0}$$

$$= c_v \log_e p_x + c_p \log_e v_x - \text{constant},$$

where  $T_x$  and  $\phi_x$  of the desired point are expressed in terms of the observed pressures and volumes as measured on the indicator card.

This computation presupposes a knowledge of  $T_0$ , but if this is unknown it can be assumed, and then the

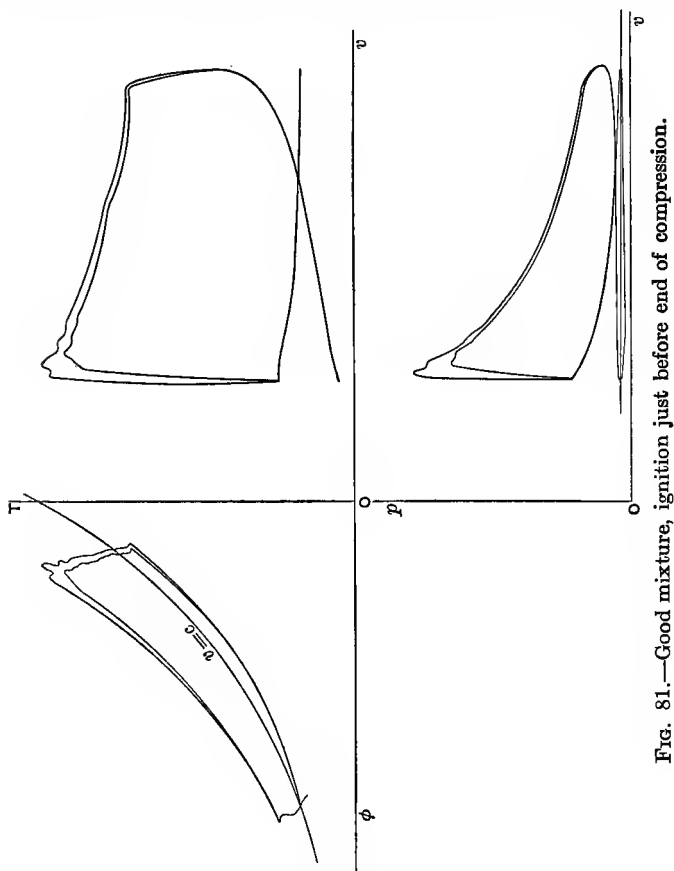


FIG. 81.—Good mixture, ignition just before end of compression.

the  $T\phi$ -plot will give relative values of  $T$  and  $\phi$  but not absolute values. As this operation must be repeated for a number of points large enough to deter-

mine the curves with sufficient accuracy, it requires much time and patience.

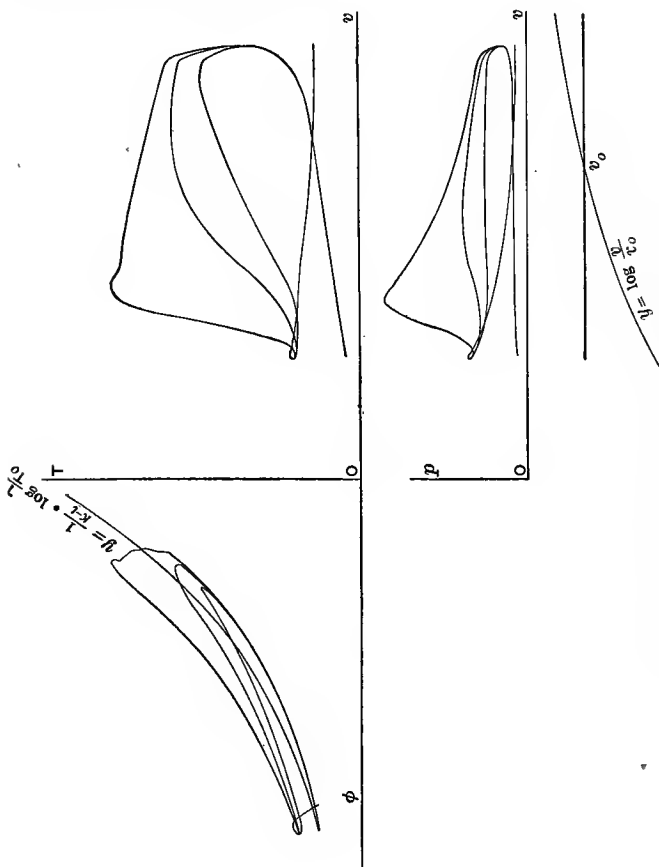


FIG. 82.—Weak explosive mixtures, late ignition.

The method of graphical projection from the  $pv$ - into the  $T\phi$ -plane, as explained in Chapter II, was used in drawing Figs. 81, 82, and 83. The indicator

cards were taken from the 36 horse-power gas-engine at the Institute: Boston illuminating gas being used as fuel. Fig. 81 represents an average card with a good

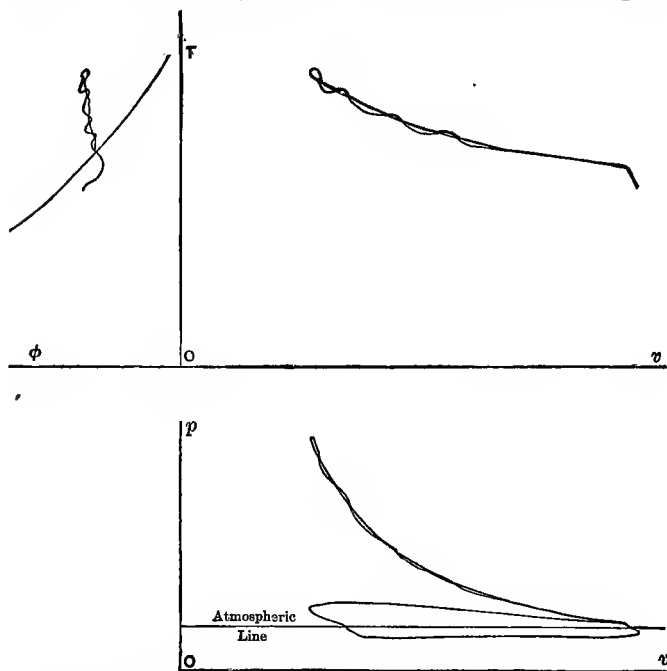


FIG. 83.—Card taken with weak spring, showing compression line, and negative work during admission and exhaust.

mixture and well timed ignition, while Fig. 82 shows the result of using a weak mixture and late ignition. Fig. 83 was taken with a weak spring to show more clearly the character of the exhaust, admission, and compression strokes. The small oscillations are chiefly due to the weak spring.

**Heat Interchanges between Gas and Cylinder.**—There is a continuous transmission of heat through the cylinder walls to the jacket water or to the air-cooling surface which may amount to from 25 per cent. to 40 per cent. of the heat of combustion. This prevents the temperature and pressure in the cylinder from ever attaining their theoretical values.

Besides this heat which is thus lost there is a second quantity which passes first to the walls and then back to the gas. We might perhaps expect to find this small compared with the condensation and re-evaporation in a steam-cylinder due to the less rapid interchange of heat between gas and metal than between wet vapor and metal, but in the gas-engine we must remember that we are dealing with much greater temperature differences.

The compression line in the  $T\phi$ -plane slants usually first to the right, showing the transference of heat from the hot metal to the cold charge. Continued compression, however, raises the temperature of the charge until it becomes equal to the mean temperature of the cylinder walls. Beyond this point the compression curve slants to the left, as the flow of heat is now from the gas to the metal. As the piston approaches the end of the compressive stroke its speed is slow, so that the rate of increase of temperature due to compression is slow, but the temperature difference between gas and metal being large the loss of heat, and hence

rate of temperature decrease, due to heat transference, is large. Thus the compression curve approaches more and more closely to the isothermal as the conduction loss approaches the compression gain. It may sometimes happen that the loss exceeds the gain and then the temperature actually decreases towards the end of compression. The magnitudes of the heat transference for any case may be determined by planimetering the areas under the respective curves and expressing in heat units.

If the ignition occurs before the end of the compression stroke, and if the combustion is not finished until after the piston has passed the dead-center, the combustion line at first approaches the constant-volume curve of the ideal cycle, becomes tangent to it at the moment dead-center is passed, and then proceeds to fall off from it until combustion is finished. Figs. 81 and 82 show the character of the combustion curve for different times of firing.

The expansion line varies widely in character, according to the percentage mixture of the charge, the time of firing, and the speed of the engine. For slow-burning mixtures (slow burning with reference to the time duration of a revolution) the line may slope continuously to the right, showing the constant addition of heat up to the moment of release (Fig. 82). Mixtures in which the combustion is finished shortly after the beginning of expansion give cards of approximately

the character shown in Fig. 81. At first the expansion line frequently shows decreasing entropy due to the rapid interchange of heat between the incandescent gas and the relatively cold metal. The inner layer of metal soon reaches the temperature of the gas and then as the temperature of the gas is decreased by further expansion the inner surface of the cylinder possesses the highest temperature and heat flows from it in two directions, outwards towards the cold jacket and inwards to the cold gas. This last change manifests itself by increasing entropy during the latter part of the expansion. Sometimes this second part of the expansion line becomes practically isothermal and thus lends credence to the theory of "after burning"—this isothermal process being apparently at the highest temperature at which recombination of the dissociated elements may occur.

The character of the exhaust line is of no significance, as it does not represent the history of a fixed quantity of substance. Its sole importance is to close the diagram and thus to make the area of the  $T\phi$ -diagram the heat equivalent of the work recorded by the indicator card.

## CHAPTER XI.

### THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO THE NON-CONDUCTING STEAM-ENGINE.

THE Carnot cycle for steam gives a very good  $pv$ -diagram, and hence there are not the same mechanical objections to its adoption as in hot-air engines. But, due to the physical change in the working fluid, a differ-

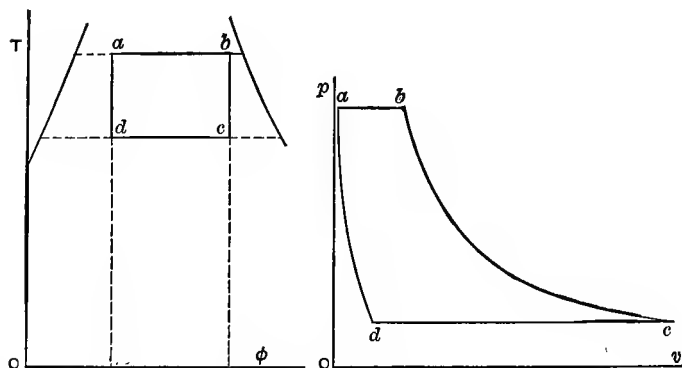


FIG. 84.

ent cycle has proved to be more feasible. In the Carnot engine the steam at condition  $d$ , Fig. 84, would be compressed adiabatically to  $a$  with the change in



condition from  $x_d$  to  $x_a$ . The isothermal expansion  $ab$  occurring by the application of heat to the cylinder produces the further change in condition to  $x_b$ . The cycle is finished by the adiabatic expansion  $bc$  and the isothermal compression  $cd$  with the cylinder in contact with the refrigerator.

The card of the ideal engine differs materially from this. (1) The line  $ab$ , Fig. 85, represents the admission

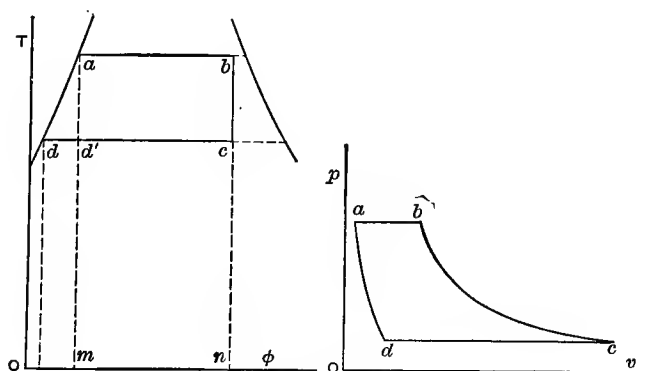


FIG. 85.

of steam of condition  $x_b$  into the cylinder up to the point of cut-off. This is forced in by the vaporization of an equivalent amount in the boiler, so that the  $T\phi$ -curve is the same as in the Carnot engine. (2) The adiabatic line  $bc$  represents the expansion of the steam admitted along  $ab$  plus the amount already in the clearance space at the moment of admission  $a$ . (3) During exhaust the piston simply forces out into the

condenser all of the steam taken during admission, but the quality of the *remaining* portion is the same at compression as at the beginning of release. (4) The part confined in the clearance space is then compressed along  $da$  in the  $pv$ -plane up to the initial pressure, i.e., back along the curve  $cb$  in the  $T\phi$ -plane. Hence, in a non-conducting engine, the amount of steam confined in the clearance space is immaterial, as its expansion and compression occur along the same adiabatic and do not affect the heat consumption.

That part of the steam exhausted during release, however, passes into the condenser and there condenses and gives up its heat to the cooling water. This is represented by  $cd$ .

From the condenser the water is forced into the boiler by means of a feed-pump, and is there warmed from  $d$  to  $a$  and vaporizes from  $a$  to  $b$ . The  $pv$ -diagram gives a history of the work performed per stroke and is confined entirely to the events in the cylinder. The  $T\phi$ -diagram, however, represents the heat cycle, and consists of events occurring in three different places.  $da$  and  $ab$  represent the heating of the feed-water and its evaporation at working pressure in the boiler,  $bc$  represents the adiabatic expansion in the cylinder of the engine, and  $cd$  the discharge of heat to the condenser.

If it were desired to make this cycle into a Carnot, the condensation would have to stop at  $d'$  and the

feed-pump arranged to compress the mixture adiabatically to  $a$ .

Suppose each engine to use one pound of steam of condition  $x_b$ , Fig. 85, per stroke, then the efficiency will be

$$\begin{aligned} a) \quad \text{Carnot:} \quad \eta &= \frac{(x_b - x_a)r_1 - (x_c - x_d')r_2}{(x_b - x_a)r_1}; \\ &= \frac{\text{area } abnm - \text{area } d'cnm}{\text{area } abnm} = \frac{T_1 - T_2}{T_1}; \end{aligned}$$

b) Non-conducting or Rankine engine:

$$\begin{aligned} \eta &= \frac{[q_1 - q_2 + (x_b - x_a)r_1] - (x_c - x_d)r_2}{q_1 - q_2 + (x_b - x_a)r_1} \\ &= \frac{q_1 + x_b r_1 - q_2 - x_c r_2}{q_1 + x_b r_1 - q_2} = 1 - \frac{x_c r_2}{q_1 + x_b r_1 - q_2}. \end{aligned}$$

An inspection of the diagram shows at once that

$$\eta_{\text{Carnot}} > \eta_{\text{Rankine}}$$

It is evident also that for any given boiler pressure, the less the amount of moisture in the steam the smaller the difference between the Carnot and the Rankine cycles.

**Increased Efficiency by Use of High-pressure Steam.**—If the same quantity of heat be supplied per pound of steam under constantly increasing pressure the state point,  $b$ , Fig. 86, will assume the successive posi-

tions  $b'$ ,  $b''$ , etc., along the constant heat curve  $bb''$ , and at the same time the state point  $c$ , representing the condition at the end of the adiabatic expansion to the back pressure, moves towards the left into the successive positions  $c'$ ,  $c''$ , etc. Now the areas under  $cd$ ,  $c'd$ ,  $c''d$ , etc., represent the quantity of heat dis-

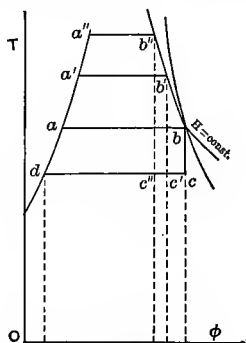


FIG. 83.

charged to the condenser under the different conditions. Therefore the greater the pressure or the higher the temperature at which a given quantity of heat is supplied to the engine, the smaller the fractional part rejected to the condenser, that is, the larger the portion turned into work and the greater the efficiency.

The  $Tp$ -curve shows that at high pressures the pressure increases much more rapidly than the temperature, and hence the necessary strength, weight, and cost of the engine will increase more rapidly than the gain in efficiency.

It should be noted, however, that there is also an upper limit to the theoretical gain in efficiency from increasing the initial pressure. Although our knowledge of the properties of saturated steam above 310 pounds pressure is inexact and largely obtained by extrapolation, nevertheless it suffices to define approximately the contour of the water and steam lines up to their junction at the critical temperature (see table on p. 388). The complete diagram for saturated steam is approximately as shown in Fig. 87. There is no doubt that for a while increasing the pressure increases the efficiency, but at the same time the heat of vaporization is diminishing with increasing rapidity (approaching zero as a limit at the critical temperature), so that, as the water and steam lines converge, the discrepancy between the Rankine and the Carnot cycles grows more and more marked. From being nearly identical at lower pressures the Rankine efficiency finally attains to only about one half the Carnot efficiency. For dry steam the point at which the Rankine efficiency reaches its maximum value appears to be at about a pressure of 2000 pounds, the exact point varying somewhat with the back pressure.

**Table of Rankine Efficiencies.**—The table of efficiencies on page 231 shows very clearly the relative gains to be expected from increase of boiler pressure or decrease of back pressure. Thus starting, for example, with an absolute boiler pressure of 70 pounds and run-

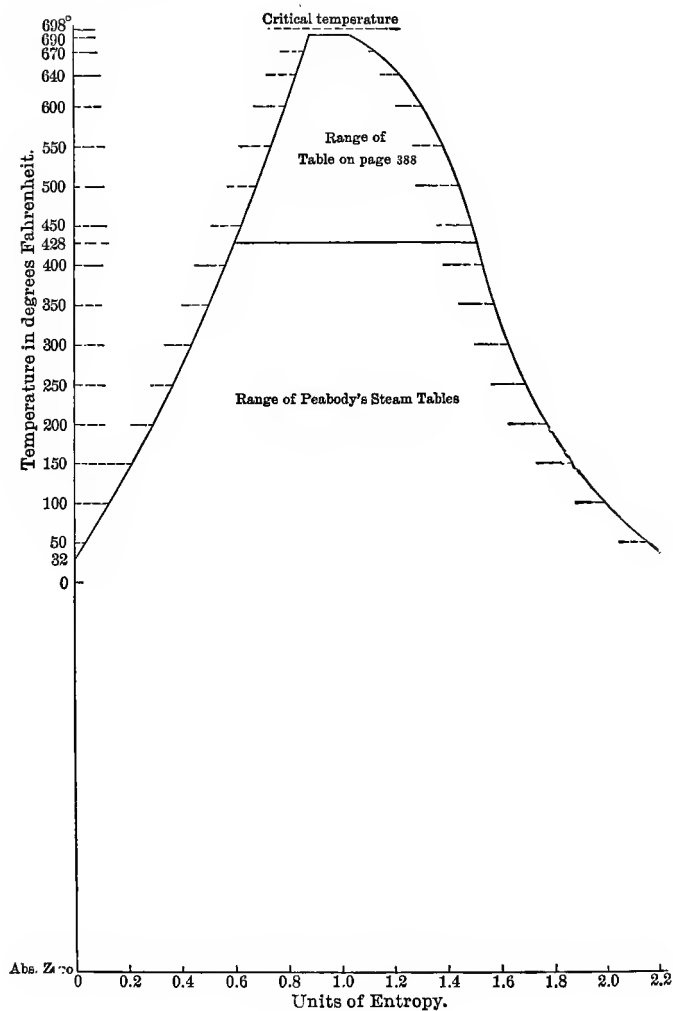


FIG. 87.

RANKINE'S EFFICIENCY WITH DRY SATURATED STEAM  
FOR DIFFERENT INITIAL AND FINAL PRESSURES.

Initial Conditions.		Back Pressures, Pounds Absolute.						
$p_1$	$t_1$	20.0	14.7	5	4	3	2	1
5.99	170	....	....	0.2	1.7	3.4	5.9	9.7
7.51	180	....	....	2.7	4.1	5.9	8.2	11.8
9.34	190	....	....	4.2	5.6	7.3	9.6	13.1
11.53	200	....	....	5.6	6.9	8.6	10.8	14.3
14.13	210	....	....	6.9	8.2	9.9	12.1	15.5
14.70	212	....	....	7.2	8.5	10.1	12.4	15.6
17.19	220	....	1.2	8.3	9.5	11.1	12.7	16.6
20.78	230	0.3	2.6	9.5	10.8	12.3	14.4	17.6
25.0	240	1.7	3.9	10.7	11.9	13.4	15.4	18.6
29.8	250	3.1	5.3	11.8	13.0	14.4	16.5	19.6
35.4	260	4.4	6.5	12.9	14.1	15.6	17.5	20.6
41.8	270	5.6	7.7	14.1	15.2	16.6	18.5	21.5
49.2	280	6.9	8.9	15.1	16.2	17.6	19.4	22.4
57.5	290	8.2	10.0	16.0	17.1	18.5	20.4	23.2
67.0	300	9.1	11.1	17.0	18.1	19.5	21.2	24.0
77.6	310	10.2	12.1	17.9	19.0	20.3	22.1	24.8
89.6	320	11.3	13.2	18.8	19.9	21.1	22.8	25.7
103.0	330	12.3	14.1	19.7	20.7	21.9	23.7	26.3
117.9	340	13.2	15.1	20.5	21.5	22.7	24.4	27.0
134.5	350	14.2	15.9	21.3	22.3	23.5	25.1	27.7
152.9	360	15.1	16.9	22.0	23.0	24.2	25.8	28.4
173.2	370	16.0	17.7	22.8	23.7	25.0	26.5	29.0
195.5	380	16.8	18.5	23.5	24.4	25.5	27.2	29.6
220.0	390	17.6	19.2	24.2	25.1	26.2	27.8	30.2
246.9	400	18.4	20.0	24.8	25.8	26.9	28.4	30.7
276.3	410	19.1	20.8	25.5	26.4	27.5	29.0	31.3
308.5	420	19.8	21.4	26.1	27.0	28.1	29.6	31.8
336.2	428	20.4	22.0	26.6	27.5	28.6	30.0	32.2
422.4	450	21.9	23.3	27.9	28.7	29.7	31.1	33.3
678.5	500	24.8	26.2	30.6	31.1	31.2	33.7	35.4
956.1	540	27.3	28.0	31.9	32.7	33.6	34.8	36.7
1212	570	28.0	29.2	32.9	33.6	34.5	35.7	37.5
1516	600	28.9	30.0	33.6	34.3	35.2	36.3	38.1
1867	630	29.5	30.6	34.1	34.7	35.5	36.6	38.4
2137	650	29.6	30.7	34.1	34.7	35.5	36.6	38.3
2431	670	29.3	30.3	33.6	34.3	35.0	36.0	37.7
2748	690	21.4	22.7	26.8	27.4	28.4	29.6	31.6
2882	698	20.8	22.0	25.6	26.3	27.1	28.2	30.1

ning non-condensing the Rankine efficiency is found to be 11.4 per cent. If the back pressure is lowered to 1 pound absolute the efficiency becomes 24.4 per cent or is more than doubled, while if the initial pressure is raised to 336.2 pounds absolute the efficiency becomes only 22.0 per cent. The combination of both changes results in an efficiency of 32.2 per cent. Thus in the case cited decreasing the back pressure 13.7 pounds produces a greater gain than that obtained by increasing the initial pressure by 266 pounds. This illustrates the great gain in efficiency to be obtained by running an engine condensing.

**The Low-pressure Turbine.** — Steam exhausted at atmospheric pressure is still capable of developing 15.6 per cent of its total heat into work if used in a turbine exhausting at 1 pound absolute, which is equal to the work to be obtained from a non-condensing engine taking steam at 128.3 pounds absolute.

To utilize to full advantage the heat in low pressure steam, a reciprocating engine would need to be exceedingly large and thus possess large friction losses, whereas a turbine is especially adapted for low pressures as a large part of its friction is due to the windage of the rotor and this diminishes as the density of the working fluid diminishes. The ideal unit for economical power production would therefore seem to be a reciprocating engine for the high pressures combined with a turbine for the low pressures, thus combining the best conditions for both.



That is, since the clearance between the blades and guides of a turbine must possess the same absolute value in all stages, the clearance area relatively to the blade area must be larger in the high pressure stages than in the low, so that from this view point alone the efficiency of the high pressure stages might be expected to be less than that of the low pressure stages. But this is just the condition under which steam can be most advantageously used in a reciprocating engine as its small volume permits of small cylinders and high mean effective pressures.

Thus a combined unit using a reciprocating engine where the turbine is least efficient and using a turbine where the reciprocating engine is least efficient, may be expected to give a higher total thermal efficiency than either type of motor used alone throughout the entire range of pressure.

**Effect of Different Substances upon the Rankine Efficiency.**—The discrepancy between the Rankine and Carnot cycles is caused by the slope of the liquid line. The larger the specific heat of the liquid the greater the growth of entropy during rise of temperature and the more the liquid line inclines from the isentropic. That is, other things being equal, the substance possessing the smallest specific heat would give the maximum value of the Rankine efficiency. (See  $L_1$  and  $L_2$ , Fig. 88.)

The effect of the sloping liquid line is further enhanced

or diminished by the relative positions of the liquid and dry vapor lines,  $V_1$  and  $V_2$ , Fig. 83. Thus if the latent heat of vaporization were doubled the discrepancy between the Carnot and Rankine cycles would for the same specific heat of the liquids be about halved, etc.

That substance possessing the smallest specific heat and the largest latent heat of vaporization will give the maximum value of the Rankine efficiency between any two temperatures. The vapor tables show that these properties are varying simultaneously, so that it becomes

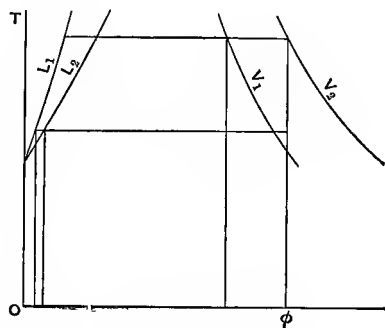


FIG. 88.

a special problem for any given temperature range to determine which of the available fluids would give the maximum Rankine efficiency.

For the engineer, however, the thermal efficiency is but one of many factors. Thus if the same power is to be developed with the different fluids a different weight of each substance must be used, so that  $w(H_1 - H_2)$  may be the same. This means a different pressure and volume range, and thus the size and strength of the

engine to be used with each fluid would need to be computed. Finally the cheapness and availability of each must be considered.

### Gain in Efficiency from Decreasing the Back Pressure.

—If the initial pressure be kept constant (Fig. 89) and the back pressure be diminished by increasing the vacuum, the heat taken up in the boiler by each pound

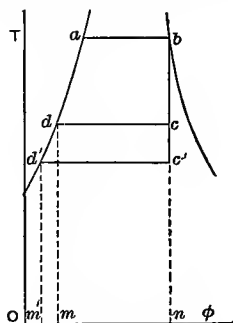


FIG. 89.

of steam will be increased from  $dabnm$  to, say,  $d'abnm'$ , and the heat discharged to the condenser will diminish from  $dcnm$  to  $d'c'nm'$ ; that is, the efficiency increases.

Again referring to the  $pT$ -curve, it is clear that at low pressure the temperature decreases much more rapidly than the pressure, so that a small decrease in pressure means a considerable increase in efficiency. This is at once evident from an inspection of the efficiency for a Carnot cycle,  $\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$ , but the expression for the Rankine cycle,

$$\eta = 1 - \frac{x_c r_2}{q_1 + x_b r_1 - q_2},$$

does not show the influence of the upper or lower temperature very clearly. The efficiency may easily be expressed as a function of the upper and lower temperatures by assuming the value of the specific heat of water to be constant and equal to unity. Thus

$$q_1 = \int_{32}^{t_1} c dt = T_1 - T_{32}, \quad q_2 = \int_{32}^{t_2} c dt = T_2 - T_{32},$$

and

$$\begin{aligned} x_b r_2 &= \left( \theta_1 - \theta_2 + \frac{x_b r_1}{T_1} \right) \cdot T_2 = \left( \log_e \frac{T_1}{T_{32}} - \log_e \frac{T_2}{T_{32}} + \frac{x_b r_1}{T_1} \right) T_2 \\ &= \left( \log_e \frac{T_1}{T_2} + \frac{x_b r_1}{T_1} \right) T_2, \end{aligned}$$

$$\text{or} \quad \eta = 1 - \frac{\left( \log_e \frac{T_1}{T_2} + \frac{x_b r_1}{T_1} \right) T_2}{T_1 - T_2 + x_b r_1}.$$

The value of the last term decreases, hence the efficiency increases,

- (1) as  $x_b$  approaches unity,
- (2) as  $T_1$  increases, and
- (3) as  $T_2$  decreases.

**Gain in Efficiency from Using Superheated Steam.—**

To avoid the introduction of excessively high pressures superheated steam is being used more and more. According to the Carnot cycle the gain in efficiency is equally great whether superheated or saturated steam of the

same temperature is used, but the Rankine cycle shows that the *theoretical* gain to be expected from superheated steam is but slight.

The portion *be* of the Rankine cycle, Fig. 90. represents the addition of heat in the superheater, and *ec* the expansion from superheated to saturated steam

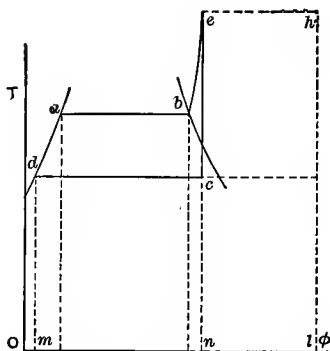


FIG. 90.

in the cylinder; the rest of the cycle is as previously described.

The heat  $q_1 - q_2 + r_1 + c_p(t_s - t_1)$  is received along the line of varying temperatures *dabe*, while in the Carnot cycle an equal quantity of heat (area *ehln* = area *dabenm*) is all received at the upper temperature  $t_s$ . Hence the efficiency of the Rankine is now much less than that of the Carnot cycle working between the same temperature limits and the discrepancy increases as the degree of superheating increases.

The analytical formulæ for this case are:

$$\eta = \frac{q_1 - q_2 + r_1 + c_p(t_s - t_1) - x_c r_2}{q_1 - q_2 + r_1 + c_p(t_s - t_1)} = 1 - \frac{x_c r_2}{q_1 - q_2 + r_1 + c_p(t_s - t_1)}$$

$$= 1 - \frac{\left( \log_e \frac{T_1}{T_2} + \frac{r_1}{T_1} + c_p \log_e \frac{T_s}{T_1} \right) T_2}{T_1 - T_2 + r_1 + c_p(T_s - T_1)} \text{ (approximately).}$$

This shows an increase in efficiency with increasing  $T_s$ , but only of small amount.

It is evident, then, that the great gain obtained by using superheated steam must be looked for in the overcoming of certain defects inherent in an actual engine. The use of steam expansively entails a cooling of the working fluid and hence of the cylinder walls containing it. This effect is increased by release occurring before the expansion has reached the back pressure, and is only partially counteracted in part of the cylinder walls by the heating effect produced during compression. Thus the entering steam undergoes partial condensation before the cylinder walls have been brought up to its temperature; that is, each pound of steam, instead of occupying the volume which it had in the steam-pipe, now occupies a reduced volume proportional to the condensation. And hence, instead of obtaining the total area  $abcd$ , Fig. 91, only the fractional part  $akld$  can be utilized. Thus the area  $kbcl$  has been subtracted from the numerator of the expression for efficiency. This condensation may

range as high as from 20 per cent. to 50 per cent. of the total steam.

The addition of superheated steam may result in the superheat *bemn* being sufficient to supply the heat taken by the cylinder walls and thus preventing the condensation and making available the area *kbcl*. The economy is further increased as the steam at the

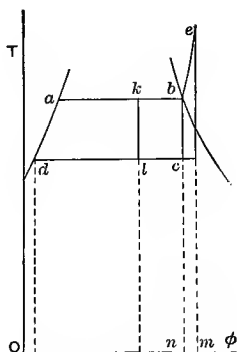


FIG. 91.

end of expansion has less moisture in it and thus abstracts less heat from the cylinder walls during release. That is, the conduction of heat through a vapor occurs but slowly, while water in contact with the metal will abstract large quantities of heat during evaporation. The leakage loss is also less with superheated steam.

**Loss in Efficiency Due to Incomplete Expansion.**—If steam be taken throughout the entire stroke the indicator-card is represented by *abed* (Fig. 92). The

drop in pressure  $be$  is equivalent to cooling at constant volume and may be represented on the  $T\phi$ -diagram by the curve of constant volume  $be$ . If the same quantity of steam be taken successively into larger cylinders, so that an increasing degree of expansion is obtained, this will be represented by  $bc$ ,  $bc'$ ,  $bc''$ , etc., in both diagrams. The areas  $B$ ,  $C$ ,  $D$  show the extra work performed per pound in the  $pv$ -plane, and the

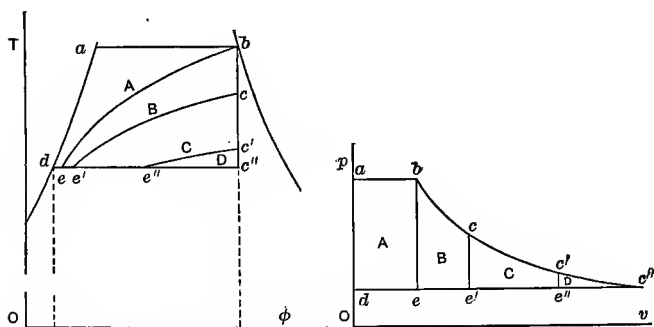


FIG. 92.

extra heat utilized in the  $T\phi$ -plane respectively, as the expansion progresses from initial to final pressure.

As in the gas-engine, so in the steam-engine it seldom pays to carry the expansion completely down to back pressure, because the slight gain from  $c'$  to  $c''$  is more than counterbalanced by the increased size, cost, and weight of the engine, friction, and radiation losses, etc.

For such incomplete expansion the expression for the efficiency of the Rankine cycle is found as follows:



$$Q_1 = q_1 - q_2 + x_1 r_1,$$

$$Q_2 = Q_{ce} + Q_{ed} = E_c - E_e + Q_{ed}$$

$$= (q_c + x_c \rho_c) - (q_2 + x_e \rho_2) + x_c r_2 + (A p_c x_c u_c - A p_e x_e u_e)$$

$$= q_c + x_c r_c - q_2 - A x_c u_c (p_c - p_2),$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{x_c r_c + q_c - q_2 - A x_c u_c (p_c - p_2)}{q_1 - q_2 + x_1 r_1}.$$

### Loss in Efficiency from Use of Throttling Governor.—

The throttling governor acts by wire drawing the steam to a lower pressure. Less steam is thus taken per stroke, as the volume is increased by both the reduced pressure and the increased value of  $x$ . A series of cards for dropping pressure is shown in Fig. 93. The

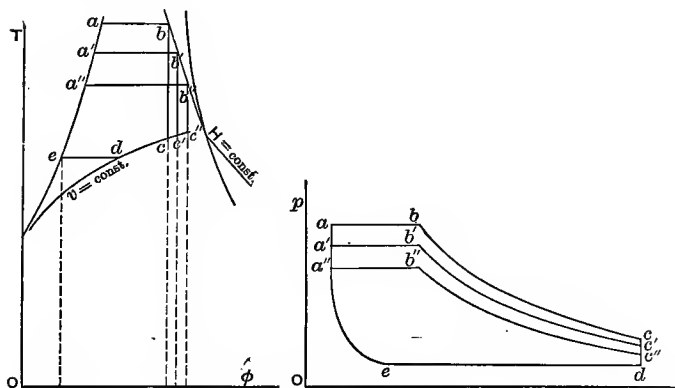


FIG. 93.

$T\phi$ -diagram shows the decreased efficiency *per pound* of steam for the same cases. During wire drawing the heat remains the same, but the entropy increases, as

the process is irreversible. The heat rejected increases as the initial pressure drops, so that of the total heat brought in a smaller quantity is changed into work and the efficiency of the plant decreased.

**Pounds of Water per Horse-power-hour for the Rankine Cycle.**—It is customary to quote the economy of an engine in pounds of water per horse-power per hour. In time one acquires a rough knowledge of the variations in water consumption for engines of different sizes under varying conditions, but this is at the best a crude method of comparison between the results of various engines unless one possesses a standard of reference and can show how closely the actual engines have approached the ideal conditions in each case. The Rankine cycle represents the thermal history of the working fluid in a plant containing no heat losses. The maximum amount of work is thus obtained from each pound of steam and, therefore, the steam consumption of the Rankine cycle represents, for any given set of conditions, the minimum amount which must be used per hour to develop a horse-power of work.

A horse-power is equal to 33,000 foot-pounds of work per minute, so that the heat equivalent of a horse-power is 2545 B.T.U. per hour. The heat utilized per pound in developing useful work is equal to the difference of the total heats at the beginning and end of the isentropic expansion. The pounds of steam required

to develop one horse-power-hour of work are therefore equal to  $\frac{2545}{H_1 - H_2}$ .

The table on page 244 shows the effect of varying initial and final pressures upon the water consumption of the ideal engine using dry steam.

**Pounds of Water per Kilowatt-hour for the Rankine Cycle.**—A kilowatt is equal to  $\frac{1000}{746}$  H.P., and is therefore equivalent to 44,240 foot-pounds of work per minute, which is equivalent to the utilization in work of 3412 B.T.U. per hour. The pounds of steam required to develop one kilowatt-hour of electrical energy are therefore equal to  $\frac{3412}{H_1 - H_2}$ .

These values could be readily tabulated if necessary but they may also be obtained by multiplying the corresponding values for pounds of steam per horse-power-hour by the factor 1.341.

**Use of Entropy Diagram or Tables.**—The labor involved in computing the ideal water consumption and the Rankine efficiency for any given case can be greatly reduced by using either a temperature-entropy chart containing constant-heat curves or Peabody's entropy tables. The method of using to obtain the water consumption is evident from the expression

$$\frac{2545}{H_1 - H_2}$$

## POUNDS OF DRY STEAM PER HORSE-POWER-HOUR FOR RANKINE CYCLE.

Initial Condition.		Back Pressures, Pounds Absolute.						
$p_1$	$t_1$	20.0	14.7	5	4	3	2	1
5.99	170	....	....	1340	150.6	72.9	42.0	25.0
7.51	180	....	....	94.3	60.8	42.3	29.6	20.2
9.34	190	....	....	59.3	44.4	33.6	25.3	18.10
11.53	200	....	....	44.8	35.8	28.5	22.4	16.52
14.13	210	....	....	36.0	30.0	24.8	20.0	15.28
14.70	212	....	....	34.7	29.2	24.2	19.5	15.08
17.19	220	....	225.2	30.2	25.8	21.9	19.0	14.20
20.78	230	908.8	99.0	26.2	22.9	19.7	16.63	13.32
25.0	240	156.2	66.1	23.2	20.6	18.04	15.46	12.55
29.8	250	86.3	49.2	20.8	18.74	16.78	14.40	11.88
35.4	260	59.9	39.4	18.9	17.24	15.48	13.56	11.28
41.8	270	46.4	33.3	17.43	15.99	14.45	12.80	10.76
49.2	280	38.5	28.8	16.19	14.92	13.61	12.14	10.30
57.5	290	32.6	25.5	15.18	14.11	12.91	11.55	9.92
67.0	300	28.4	23.0	14.29	13.33	12.24	11.07	9.57
77.6	310	25.3	21.0	13.52	12.66	11.69	10.61	9.24
89.6	320	22.9	19.3	12.84	12.06	11.20	10.22	8.90
103.0	330	20.9	17.93	12.26	11.57	10.78	9.85	8.67
117.9	340	19.4	16.78	11.75	11.10	10.40	9.52	8.43
134.5	350	18.05	15.83	11.27	10.68	10.02	9.24	8.20
152.9	360	16.96	14.93	10.87	10.32	9.71	8.96	7.97
173.2	370	15.95	14.19	10.47	10.00	9.40	8.72	7.81
195.5	380	15.18	13.54	10.16	9.69	9.15	8.49	7.63
220.0	390	14.42	12.99	9.83	9.40	8.90	8.29	7.46
246.9	400	13.80	12.46	9.57	9.15	8.68	8.09	7.32
276.3	410	13.25	12.00	9.32	8.93	8.48	7.92	7.18
308.5	420	12.74	11.60	9.08	8.71	8.29	7.76	7.05
336.2	428	12.38	11.30	8.91	8.55	8.14	7.64	6.95
422.4	450	11.4	10.55	8.43	8.12	7.76	7.31	6.69
678.5	500	10.1	9.46	7.73	7.52	7.43	6.85	6.31
956.1	540	9.58	9.00	7.52	7.29	7.01	6.66	6.18
1212	570	9.37	8.83	7.45	7.23	6.97	6.65	6.175
1516	600	9.40	8.87	7.54	7.32	7.06	6.74	6.28
1867	630	9.71	9.19	7.82	7.61	7.35	7.02	6.54
2137	650	10.2	9.96	8.23	8.00	7.73	7.38	6.88
2431	670	11.2	10.6	9.89	8.76	8.46	8.07	7.50
2748	690	18.1	16.7	13.2	12.7	12.1	11.3	10.3
2882	698	25.4	23.2	18.2	17.4	16.5	15.4	13.9

To obtain the Rankine efficiency we have

$$\eta = \frac{q_1 - q_2 + x_1 r_1 - x_2 r_2}{q_1 - q_2 + x_1 r_1} \quad \text{or} \quad \frac{q_1 - q_2 + r_1 + c_p(T_s - T_1) - x_2 r_2}{q_1 - q_2 + r_1 + c_p(T_s - T_1)},$$

which may be regrouped and written in the form,

$$\eta = \frac{H_1 - H_2}{H_1 - q_2},$$

in which all the quantities may be read directly from the chart or the tables.

**Heat Consumption of the Rankine Cycle in B.T.U. per Horse-power-minute.**—It is customary to rate the economy of an engine in terms of the heat units required to produce one indicated horse-power per minute. As a standard of comparison the heat consumption of the Rankine cycle using dry saturated steam is given in the table on page 246.

The heat equivalent of one horse-power is 42.42 B.T.U. per minute. The heat required by the engine per minute may be determined by dividing the heat utilized by the thermal efficiency, therefore,

$$\text{B.T.U. per I.H.P. per minute} = \frac{42.42}{\eta}.$$

Or, again, the heat supplied to the engine per horse-power-hour equals the pounds of steam per horse-power-hour multiplied by the heat absorbed per pound of water in the boiler; therefore,

$$\text{B.T.U. per I.H.P. per minute} = \frac{\text{Lbs. steam} \times (H_1 - q_2)}{60}.$$

B.T.U. PER HORSE-POWER-MINUTE FOR RANKINE  
CYCLE USING DRY STEAM.

Initial Conditions.		Back Pressure, Pounds Absolute.						
$p_1$	$t_1$	20.0	14.7	5	4	3	2	1
5.99	170	....	....	22170	2516	1232	720	439.3
7.51	180	....	....	1582	1031	724	515	359.5
9.34	190	....	....	1000	755	578	441.3	323.4
11.53	200	....	....	758	611	492.5	392.5	296.9
14.13	210	....	....	611	514	429.3	351.4	274.3
14.70	212	....	....	590	500	420.0	342.6	271.4
17.19	220	....	3652	514	444.4	380.8	335.0	256.4
20.78	230	14550	1612	448	394.8	344.5	294.4	241.4
25.0	240	2508	1079	397	356.6	315.9	274.8	228.0
29.8	250	1391	807	359	325.6	294.7	262.6	216.5
35.4	260	969	648	330	300.5	272.9	242.5	206.3
41.8	270	752	549	302	279.4	255.5	229.5	197.4
49.2	280	619	477	281.3	262.2	241.3	218.4	189.5
57.5	290	520	423	264.6	247.7	229.5	208.3	182.9
67.0	300	466	382	249.6	235.1	218.1	200.1	176.8
77.6	310	416	349.5	236.8	223.9	208.8	192.3	171.2
89.6	320	377	322.5	225.6	213.7	200.7	185.7	165.3
103.0	330	345	300.5	215.8	205.5	193.6	179.3	161.4
117.9	340	321	281.9	207.3	197.7	187.4	173.7	157.1
134.5	350	299	266.8	199.4	190.5	180.7	169.0	153.2
152.9	360	282	251.9	192.6	184.5	175.4	164.2	149.1
173.2	370	266	240.1	186.0	179.1	170.0	160.1	146.5
195.5	380	253	229.4	180.7	173.8	166.2	156.1	143.3
220.0	390	241	215.6	175.2	168.9	161.6	152.7	140.4
246.9	400	231	211.9	170.8	164.6	157.9	149.6	138.0
276.3	410	222	204.5	166.5	160.9	154.4	146.3	135.6
308.5	420	214	197.9	162.5	157.0	151.2	143.5	133.3
336.2	428	208	193.1	159.7	153.9	148.6	141.6	131.6
422.4	450	193.7	181.8	152.3	147.8	142.8	136.3	127.4
678.5	500	170.8	162.1	138.9	136.3	136.1	126.0	119.7
956.1	540	158.8	151.7	132.9	129.9	126.5	121.8	115.4
1212	570	151.8	145.5	128.8	126.2	123.0	119.1	113.2
1516	600	146.9	141.1	126.2	123.7	120.6	114.2	111.4
1867	630	143.7	138.6	124.5	122.2	119.4	116.0	110.6
2137	650	143.2	138.0	124.5	122.3	119.5	116.1	110.9
2431	670	145.0	139.9	126.4	123.8	112.3	117.8	112.6
2748	690	198.6	187.0	159.0	154.6	149.5	143.2	134.2
2882	698	203.4	192.7	165.8	161.6	156.6	150.2	141.1

The heat consumption may therefore be determined from either of the preceding tables, the results are as shown on page 246.

**B.T.U. per Kilowatt-minute.**—The heat equivalent of one kilowatt is 56.9 B.T.U. per minute. The heat required by the engine per minute to produce one kilowatt when working upon the Rankine cycle is found by dividing 56.9 B.T.U. by the thermal efficiency. Or the same value may be obtained by multiplying the corresponding value for B.T.U. per horse-power-hour taken from the table on page 246 by the factor 1.341.

## CHAPTER XII.

### THE MULTIPLE-FLUID OR WASTE-HEAT ENGINE.

IN the discussion of the Rankine cycle it was shown how the efficiency of the steam-engine could be increased by raising the temperature of the source of heat or by decreasing that of the refrigerator. Due to the course of the  $pT$ -curve a practical upper limit is soon reached in the use of saturated steam due to the rapid increase of pressure at upper temperatures, so that recourse has to be taken to superheated steam. Again, in reducing the back pressure a slight drop in pressure means a large drop in the exhaust temperature, but a practical limit is soon reached beyond which it does not pay to carry a vacuum.

Theoretically, at least, the efficiency could be increased by using for the higher temperatures some fluid ( $X$ ) having a smaller vapor pressure than saturated steam, and for lower temperatures some fluid ( $Y$ ) having a greater vapor pressure than saturated steam at the same temperature. That is to say, the first fluid  $X$  could in a saturated condition be heated to the temperature now common for superheated steam, and then be



allowed to expand in a cylinder down to some lower temperature at which the pressure of saturated steam, would not be excessive. The surface condenser for this  $X$ -fluid would be at the same time the boiler for the steam. After the steam had expanded through two or three cylinders, it in turn would exhaust into a second surface condenser, which would be the boiler for the next fluid,  $Y$ , in the series. Thus the working substance in each case is condensed at the temperature of its own exhaust and fed back to its own boiler at this temperature. The heat of vaporization which the first fluid rejects must warm up the second as it is fed into the boiler and then vaporize part of it, so that

heat rejected = heat received,

$$\text{or} \quad xr_{\text{exhaust}} = (q + xr)_{\text{boiler}} - q_{\text{condenser}}.$$

For the sake of simplicity suppose at first that two such fluids  $X$  and  $Y$  as described could be found, and further that their liquid and saturated-vapor lines coincided in the  $T\phi$ -plane with those for water, but that the  $Tp$ -curves are entirely different.

Suppose, further, that  $p_1$  and  $p_2$  (Fig. 94) represent respectively the highest pressure which it is convenient to use, and the lowest pressure which can be obtained in a vacuum. The range of temperatures when saturated steam alone is used is limited between  $t_1$  and  $t_2$ . If, however, the fluid  $X$  is first used the upper temperature can be raised to  $t_x$  for the same maximum pres-

sure  $p_1$ . Suppose in this problem that the substance  $Y$  does not solidify at  $32^\circ \text{F.}$ , its liquid line would extend to the left of the arbitrary zero for the entropy of water, and so the expansion of this fluid down to  $p_2$  would drop the lower temperature from  $t_2$  down to  $t_Y$ .

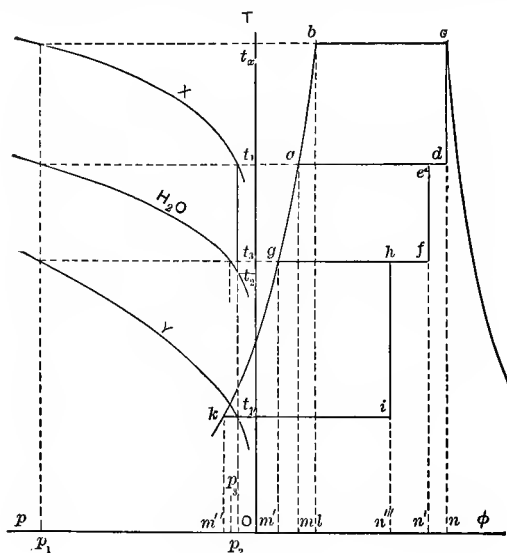


FIG. 94.

The liquid  $X$  is fed into its boiler at the temperature  $t_1$ , and is warmed along  $ab$ , receiving the heat  $q_b - q_a$ , equal to area  $ablm$ ; it is then vaporized at the pressure  $p_1$  and receives the heat  $r_X$  equal to  $bcnl$ . Its ideal cycle is now completed by adiabatic expansion  $cd$  down to some pressure  $p_X$  (in Fig. 94  $p_X$  coincides with  $p_2$ ), corresponding to temperature  $t_1$ , on the  $X$ -

curve, and condensation along  $da$ . The condensed fluid is at the proper temperature to be returned to its boiler.

The heat rejected along  $da$ , viz.,  $x_d r_d$ , must warm up the water fed into the  $X$ -condenser at temperature  $t_3$  up to  $t_1$  and then vaporize part of it at the upper temperature. Assuming no heat lost,

$$\text{area } m'gaen' = \text{area } madn.$$

The steam describes the ideal cycle  $gaef$ , rejecting in turn the heat under  $fg$ , equal to  $x_f r_f$  at some pressure  $p_3$  corresponding to temperature  $t_3$ .

In the steam-condenser the fluid  $Y$  is first warmed up from the temperature  $t_Y$ , corresponding to  $p_2$ , to  $t_3$  and then enough vaporized to make

$$\text{area } m''kghn'' = \text{area } m'gfn'.$$

From the diagram the following conclusions can be drawn:

Heat received from fuel equals  $mabcn$ .

Heat utilized by  $X$ -engine equals  $abcd$ .

Efficiency of  $X$ -engine,  $\eta_X = \frac{abcd}{mabcn}$ .

Heat rejected by  $X$ -engine = area  $madn$  = area  $m'gaen'$   
= heat absorbed by steam-engine.

Heat utilized by steam-engine equals  $gaef$ .

Efficiency of steam-engine  $\eta_{st} = \frac{gaef}{m'gaen'}$ .

Efficiency of  $X$ - and steam-engines combined equals

$$\eta_{(X+st)} = \frac{abcd + gaef}{mabcn}.$$

Heat received by  $Y$ -engine = heat rejected by steam-engine =  $m''kghn'' = m'gfn'$ .

Heat utilized by  $Y$ -engine =  $kghi$ .

Heat rejected by  $Y$ -engine =  $m''kin''$ .

Efficiency of  $Y$ -engine  $= \frac{kghi}{m''kghn''}$ .

Efficiency of all three engines together,

$$\eta_{(X+st+Y)} = \frac{abcd + gaef + kghi}{mabcn}.$$

The heat rejected has been reduced from

$$madn \text{ to } m''kihfednm''.$$

The great gain in efficiency shown in this assumed case is deceptive. The exhaust temperature has been taken far below freezing. This could not be done unless some cooling mixture could be employed in order to condense the exhaust fluid  $Y$ .

This would be expensive and probably represent as great an expenditure of work as the increased gain

recorded by the  $Y$ -piston, possibly more. In practice the lowest temperature  $t_Y$  will be governed by that of the cheapest available condensing substance, i.e., the temperature of the cooling water at the power-station.

The upper temperature  $t_X$  will be governed by the materials used in construction.

Since  $x_{ex}r_{ex} = q_B - q_C + x_B r_B$ , it follows that  $x_B < x_{ex}$ ; that is, the value of  $x$  gradually grows smaller for succeeding fluids, so that initial condensation must be increasing in the successive cylinders. The gain which accrues from the use of superheated steam might for similar reasons be expected from the use of the superheated vapors of the  $X$ - and  $Y$ -fluids. It thus would undoubtedly pay to superheat each vapor as it leaves its respective boiler an amount sufficient to overcome the initial condensation. This might be effected by the use of a separately fired superheater suitably situated, or perhaps more economically still the hot flue gases from the first boiler furnace might be made to pass successively through all the superheaters and thus the total economy increased two ways at once.

Several different multiple-fluid engines have been proposed, usually of the binary type. When it comes to a discussion of any particular combination the actual  $T\phi$ -diagram must be drawn, and the liquid- and saturated-vapor lines will no longer be super-

imposed as was assumed in the ideal case just discussed. However, the principles already enunciated will make the application clear.

Any fluid having a low boiling-point, such as ammonia, chloroform, sulphur dioxide, ether, carbon disulphide, etc., is available for such work. All such volatile fluids possess latent heat of vaporization of small magnitude, and the smaller this is, the more volatile the substance and the greater its specific pressure at a given temperature. This leads to the practically important fact that to perform a given amount of work a greater quantity of the volatile substance must be supplied, the amount necessary increasing with the diminution of the latent heat of vaporization.

Du Trembley used a steam-ether engine. Ether superheats during adiabatic expansion and thus seems especially adapted to such work, as this would tend to prevent the excessive cooling of the cylinder during exhaust and thus do away with the losses incident to initial condensation. Fig. 19, although not drawn to accurate scale, gives an approximate idea of the relative values of the latent heat of vaporization. In round numbers the entropy as liquid and as vapor compares with water as follows:

$$\text{Water} \left\{ \begin{array}{ll} 32^{\circ} \text{ F.} \dots \theta = 0, & \theta + \frac{r}{T} = 2.18; \\ 248^{\circ} \text{ F.} \dots \theta = 0.365, & \theta + \frac{r}{T} = 1.70; \end{array} \right.$$

$$\text{Ether} \left\{ \begin{array}{l} 32^{\circ} \text{ F. } \theta = 0, \quad \theta + \frac{r}{T} = 0.34; \\ 248^{\circ} \text{ F. } \theta = 0.205, \quad \theta + \frac{r}{T} = 0.385; \end{array} \right.$$

and as the latent heat is about one sixth that of water it follows that about six pounds of ether will be required to cool each pound of steam, so that a combined  $T\phi$ -diagram might be drawn for one pound of water and six of ether.

Perhaps the most accurate and elaborate series of experiments on any binary engine was made by Prof. Josse of Berlin. He used sulphur dioxide for the secondary fluid. The best results obtained were 11.2 pounds of steam per horse-power for the steam-engine alone and the equivalent of but 8.36 pounds per horse-power per hour using the combined engine. In the test the "waste-heat" engine added 34.2 per cent. to the power obtained from the primary engine.

The steam had a pressure of 171 pounds absolute and was superheated to  $558^{\circ} \text{ F.}$  The back pressure on the low-pressure cylinder of the steam-engine was about 2.9 pounds absolute, corresponding to  $140^{\circ} \text{ F.}$  The  $\text{SO}_2$  cylinder received vapor under pressure of 143 pounds and exhausted at 48.2 pounds absolute, corresponding to a temperature of  $67.8^{\circ} \text{ F.}$

In round numbers the latent heat of  $\text{SO}_2$  is about one seventh that of water, so that it would require the

vaporization of from 6 to 7 pounds of  $\text{SO}_2$  to condense 1 pound of steam.

Fig. 95 shows the ideal cycle for a binary engine of this type working between the pressures and tem-

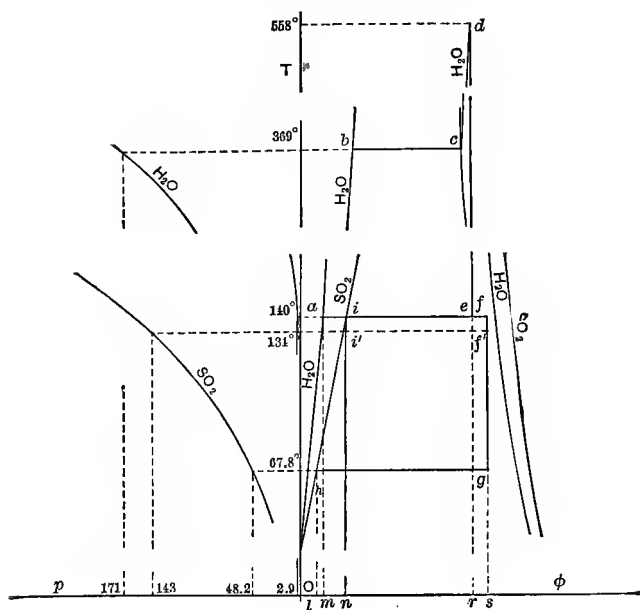


FIG. 95.

peratures realized by Prof. Josse in the test at Charlottenburg.

The steam-engine was triple expansion, and the ideal cards for such an engine are shown combined at  $abcde$ . The heat exhausted to the steam-condenser, or  $\text{SO}_2$  boiler,  $xr$ , equals  $aerm$ . Of this quantity an



amount *hifg* is saved by the SO<sub>2</sub> cylinder theoretically. Actually there was a drop in temperature of about 6° F. between the low-pressure steam-cylinder and the SO<sub>2</sub> cylinder, so that the heat area *iff'v* was either totally lost or partially reduced to lower efficiency by wire-drawing.

Dr. Schreber, in *Die Theorie der Mehrstoffmaschinen*, proposes the following combination of fluids:

Substance.	Temperature range.
\ Aniline . . . . .	590° F.—374° F.
Water . . . . .	374° F.—176° F.
Æthylamine . . . . .	176° F.—86° F.

Josse, in *Neuere Wärmekraftmaschinen* (Berlin, 1905), publishes reports of tests on several H<sub>2</sub>O-SO<sub>2</sub> engines. The SO<sub>2</sub>-cylinders developed from 450 H.P. to 150 H.P., and had been in operation from one to two years. The results show that the machines have come up to the guarantee of the manufacturers. Josse states that in view of the high initial cost such machines should probably not be installed unless the plant is to run a large number of hours daily, and unless a sufficient amount of cold cooling water is available.

## CHAPTER XIII.

### THE TEMPERATURE-ENTROPY DIAGRAM OF THE ACTUAL STEAM-ENGINE CYCLE.

THE Rankine cycle is based upon the following assumptions:

- (1) Non-conducting cylinder walls and piston;
- (2) Isentropic expansion to the back pressure;
- (3) Instantaneous action of the valves;
- (4) No leakage by the piston and the valves.

From the first two conditions it follows that the size of the clearance space is immaterial.

Referring to the actual steam-engine, we find that the conductivity of the metal produces initial condensation and reevaporation losses, and that the expansion can be carried to back pressure only by reducing the efficiency. The size of the clearance must therefore be considered, because the cycle of the clearance steam will affect the economy. The valves are not instantaneous in action, and leakage always occurs by both piston and valves. The Rankine cycle is thus unattainable in practice and is but an ideal which the actual engine strives to approximate.

The amount of condensation and reevaporation is the result of so many factors, that to determine the influence of each by the ordinary methods of comparison would require too much time and money. Hence to aid in evolving a theory of the steam-engine which shall account for all heat losses and interchanges, some convenient form of analysis must be adopted by which the losses for any single test may be investigated.

Hirn's analysis makes possible the determination of the net heat changes occurring between admission and cut-off, cut-off and release, release and compression, and compression and admission, but does not give information as to the actual direction of heat-transference at any moment. Fortunately, the  $T\phi$ -diagram offers a graphical solution equivalent to that of Hirn's analysis, and also makes clear the direction in which the interchange of heat is occurring at any point. Before a  $T\phi$ -projection of an indicator-card can be made, it will be necessary to discuss at length the different lines of the card in order to determine exactly what each represents.

**The Admission Line of the Indicator-card.**—During admission the steam is not at a uniform temperature and pressure. Part is still in the steam-pipe under boiler pressure, part has passed through the valve-chest and steam-ports, and has already entered the cylinder, and still a third portion is in the process of transition. In general, the surrounding metal is colder than the

steam, so that a continual loss of heat is experienced resulting in condensation and decrease of volume and entropy. To this is added the further effect of wire-drawing, due to too small steam passages and to the throttling effect when the valve is opening and closing, producing a drop of pressure and increase of both specific volume and entropy. It is probable that each particle of steam follows its own path in passing from the steam-pipe into the cylinder up to the point of cut-off. Thus the admission line of the indicator-card is not the  $pv$ -history of the entire quantity of steam nor of any particular part of it, and is only a record of the pressure exerted from moment to moment by the varying quantity of steam confined in the cylinder. Hence in projecting the admission-curve into the  $T\phi$ -plane it must be remembered that the projection does not represent the  $T\phi$ -history of any portion of the steam, but is simply a reproduction of each individual point of the  $pv$ -curve.

Let  $a'c'$  (Fig. 96) represent the  $T\phi$ -projection of the admission line of an indicator-card, while  $b$  represents the state point of the steam in the steam-pipe. But for the various losses the admission line would have been  $ab$ , which represents the actual path followed by the steam in the boiler. If, for a moment, we consider the admission to represent a reversible process, the area under  $a'c'$  will represent the heat received during this process. Hence the area  $abb_1c_1c'a'$  represents the

difference between the heat contained per pound of steam in the boiler, and the amount realized per pound in the cylinder, or the losses due to initial condensation and wire-drawing. The former would simply result in the condensation of part of the steam, thus causing the value of  $x$  to diminish from  $b$  to  $c$ , or possibly to some point slightly to the left of  $c$ ; the latter would cause a

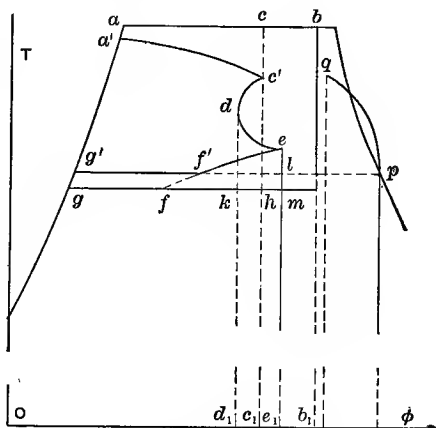


FIG. 96.

drop in pressure and increase of entropy, moving the state point to  $c'$ . Due to the impossibility of distinguishing accurately between these two opposing factors, one tending to decrease, the other to increase the entropy, the area  $cbb_1c_1$  is taken to represent the loss due to initial condensation, and the area  $acc'a'$  to represent the loss or reduction in efficiency due to friction and wire-drawing.

**The Expansion Line of the Indicator-card.**—If we assume that the leakage by piston and valves is negligible during expansion, the expansion-curve between cut-off and release represents the continuous  $pv$ -history of the entire quantity of steam contained in the cylinder; that is, of the cylinder feed plus the clearance steam. The temperature of the steam throughout the cylinder is not uniform, as heat-conduction is occurring between the steam and the metal, so that the indicator records but the average pressure due to these variable temperatures. Hence the  $T\phi$ -projection will give but average values of the  $T\phi$ -changes during expansion.

Since there is no appreciable friction of the steam against the metal, as during admission, it follows that neglecting the inequalities in the temperature of the steam, there is no reduction of the heat efficiency of the steam due to *internal* irreversible processes, and thus any increase or decrease of the entropy of the steam must result from heat-transferences between the steam and the surrounding metal. If adiabatic, the curve would here be isentropic, but as the steam is at first hotter than the cylinder walls, the flow of heat is from steam to metal, thus causing an increase in the entropy of the metal and a decrease in that of the steam.

The expansion line thus assumes at the start some such form as  $c'd$ , becoming steeper as the temperature drops, and just at the moment the temperatures of the

steam and the walls are the same it becomes isentropic. From this point,  $d$ , on to release at  $e$ , the heat transfer is from metal to steam, so that the entropy of the latter now increases and the curve slants to the right.

The ideal engine, supplied with steam of condition  $c'$ , would expand isentropically along  $c'd_1$  to the back pressure at  $h$ . Hence the area  $c'dd_1c_1$ , under the first part of the expansion-curve, represents the loss of heat due to conduction. Again, the ideal engine, supplied with steam of condition  $d$ , would expand isentropically to  $k$ . The area under  $de$ , for the actual engine, thus represents a gain due to the heat returned by the walls. It should be noted that the heat thus regained is restored at a lower temperature than that at which it was lost, and hence at a lower efficiency.

**The Exhaust Line of the Indicator-card.**—Let us consider first the case where the expansion is carried down to back pressure. The ideal engine, supplied with steam of quality  $e$ , would expand along  $ee_1$  down to the pressure in the condenser and then condense along  $mg$ . The actual engine, due to the resistance of the exhaust ports, etc., would expand to some pressure, as  $l$ , greater than that in the condenser and would then exhaust along  $lg'$ . The area  $lg'gm$  would thus represent the loss of heat due to throttling during exhaust.

If the release-valve opens at  $e$  before back pressure is reached, the phenomena are as follows: As the valve starts to open steam begins to escape and is throttled

down to the condenser pressure; as the valve continues to open the escape becomes more rapid until the back pressure is established. Then on the return stroke more steam is forced out against the back pressure, and near the end, as the exhaust-valve begins to close, there is a slight rise in pressure and a small quantity escapes, suffering reduction in efficiency by throttling. As the valve closes, all the cylinder feed has escaped, and only the clearance steam remains. It is necessary to note that the exhaust line of the card records the pressure of the steam still in the cylinder at any moment and gives no information whatever as to its condition, or of the condition of that portion already exhausted. Thus part of the steam has already reached the condenser (or in the case of a multiple-expansion engine the following cylinder or intermediate receiver), and has already parted with some of its heat, while that still in the cylinder, being at a lower temperature than the cylinder walls, is receiving heat and losing its moisture and may sometimes at compression have become even superheated. The last part of the exhaust-steam will necessarily have to retrace part of this thermodynamic process on reaching the condenser, or upon mingling with the rest of the steam in the following receiver or cylinder.

The exhaust line of the card does not represent the  $p$ - $v$ -history of any definite quantity of steam, but is simply a pressure record of continually varying quantities



confined in a constantly diminishing volume. It does, however, represent the amount of work required to discharge the steam, and in that sense the area under its  $T\phi$ -projection will represent the total heat discharged.

In the case of the ideal engine, the exhaust line,  $efg$ , divides into two parts,  $ef$  and  $fg$ , equivalent to decrease of pressure at constant volume and to decrease of volume at constant pressure respectively. The heat rejected is represented by the total area under  $efg$ , and exceeds that rejected after complete expansion to the back pressure by  $efm$ , which thus represents the extra loss incurred by incomplete expansion. The exhaust line for the actual indicator-card will be some such curve as  $ef'g'$ , where the area  $ef'l$  shows the loss due to incomplete expansion, and  $lg'gm$  the loss due to throttling, friction, etc.

**The Compression Line of the Indicator-card.**—The compression-curve, from the closing of the exhaust-valve up to the moment of admission, gives the  $pv$ -history of the clearance steam, and, if no leakage is assumed, the  $T\phi$ -projection will thus be the actual  $T\phi$ -history of a definite quantity of steam. As the pressure increases the curve deviates more and more rapidly from the adiabatic, due to the increasing effect of conduction losses, and on some cards may become nearly isothermal. In such cases it is probable that the assumption of dry steam at compression is incor-

rect, the presence of moisture helping to explain the rapid loss of heat.

During the interval between the opening of the admission-valve and the attainment of initial pressure the time interval is so small that probably the assumption of adiabatic compression of the clearance steam would not be greatly wrong. The gain in heat thus incurred must be at once lost by condensation during the first part of the admission, but it is impossible to determine the history of this change.

To obtain the  $T\phi$ -projection of the compression-curve, the saturation-curve for the weight of clearance steam should be drawn through the point of compression (assuming dry steam at compression) and the projection performed as previously described. The curve will assume some such form as  $pq$ , which may or may not, according to circumstances, have its course partially or wholly in the saturated or superheated regions. In any case the area under the curve, when reduced to the proper ratio, shows the heat lost to the walls during compression, and, if the horizontal line of the indicator-card is established at  $q$ , gives a general idea of the temperature of the cylinder at the moment of admission, and hence a measure of the heat necessary to bring the cylinder up to the temperature of the entering steam. Except for this one feature, the cycle of the clearance steam is unimportant, as all the losses occasioned by it will be manifested by the

difference between the cycle for an ideal engine, working between the given pressures in the steam pipe and condenser, and the  $T\phi$ -plot of the actual card.

**The Indicator-card.**—Considered as a whole, the indicator-card furnishes the following information. The admission line and the exhaust line simply represent the pressure of part of the steam, but do not give any information regarding the specific volume. On the other hand, both the expansion and the compression lines give the history of definite quantities of steam. Thus both the expansion and compression of the clearance steam are recorded, while only the expansion of the cylinder feed appears, the compression of the latter occurring in the boiler. The entire card, if plotted directly, can at the best be considered only as the heat equivalent of the work done, but not as the  $T\phi$ -history of any closed cycle. Thus, for example, while the projection of the exhaust line gives some such curve as  $ef'g'$ , which on the  $T\phi$ -chart indicates condensation, it is probable that the value of  $x$  of the confined steam is actually increasing.

The difficulties involved in the proper interpretation of the irreversible portions of the indicator-card have led different investigators to make certain assumptions as to the influence of the clearance steam and as to the possibility of replacing the actual curves by equivalent reversible processes.

**The Clearance Steam Considered as an Elastic Cushion.**

—During expansion the clearance steam follows the same laws and variations as the cylinder feed, and this in general is not the reverse of its history during compression. Thus the cycle of the clearance steam, if it could be drawn, would enclose an area representing either positive or negative work. This cycle would then be of especial interest in determining losses experienced by the clearance steam, but as these losses must eventually be charged against the entering steam, the total effect upon the efficiency would be the same if the clearance steam were considered as an isolated elastic cushion expanded and compressed along the same adiabatic. If, then, an adiabatic is drawn through the point of compression on the indicator-card, the horizontal distance from any point on this adiabatic to the corresponding point on the indicator-card shows the volume which the cylinder feed would occupy under the above assumptions. Taking this adiabatic as the line of zero volume, a diagram can thus be constructed which shows only the variations of the cylinder feed. It is then only necessary to draw on the saturation-curve for the weight of steam fed to the cylinder per stroke and the card can be at once projected into the  $T\phi$ -plane. This is, in its essential points, the method adopted by Prof. Reeve in his book on the *Thermodynamics of Heat Engines*, although he changes volumes so that the reconstructed card represents

the  $pv$ -history of one pound of cylinder feed instead of the actual weight in the cylinder. To project his reconstructed card into the  $T\phi$ -plane, it is only necessary to draw the saturation-curve for one pound of steam instead of that for the pounds fed per stroke. Inasmuch as the  $T\phi$ -projection will be the same in either case it seems somewhat simpler to adopt the first method, viz., to construct the saturation-curve for the number of pounds in the cylinder rather than to redraw the diagram to correspond to one pound of cylinder feed.

This method undoubtedly makes possible a determination of the general magnitude and character of the various heat interchanges, but is open to the following objections.

The compression line of the card refers to the clearance steam alone, so that the deviations from the adiabatic thus obtained refer to itself and not to the cylinder feed. Furthermore, the reconstructed curve may actually pass to the left of the water line, assuming imaginary values on the  $T\phi$ -plane, and thus give a wrong conception of the condition of this steam, which, instead of being wet, is usually dry or superheated, and lies to the right of the expansion line. Again, the expansion line no longer represents the actual  $pv$ - or  $T\phi$ -history of the steam, but an imaginary history which the cylinder feed might have if the clearance steam expanded adiabatically. The entire card thus becomes

to a certain extent imaginary, and in so far is undesirable.

**The Indicator-card Considered as a Reversible Cycle.**

—The area of the card gives the heat changed into work, and this same result may be attained by assuming the clearance steam and cylinder feed to remain in the cylinder, being heated and cooled by external means and thus caused to expand and contract along a reversible cycle coincident in shape with the actual card. The original card may thus be projected into the  $T\phi$ -plane as soon as the saturation-curve for the total weight of steam has been drawn on it. The expansion line represents the actual history of the substance, but the compression line is entirely imaginary.

This method was adopted by Prof. Boulvin in his book, *The Entropy Diagram*, from which the following two illustrations are copied, with but slight alterations. *ABCDEF*, in Fig. 97, represents the  $T\phi$ -projection of a certain indicator-card. The line *DE* represents the actual expansion history of the total steam; the other lines give more or less imaginary values. The small diagram at the right is used to interpret the compression line *AB*. If *W* and *w* represent the pounds of cylinder feed and clearance steam respectively per revolution, the large diagram represents the cycle of  $W+w$  pounds of the mixture, while the small diagram represents the entropy of *w* pounds. Assuming dry steam at compression, the heat rejected by *w* pounds

during compression is shown by area  $abb_1a_1$ . The entropy  $a_0a = A_0A$  is that due to the vaporization of  $w$  pounds of water, so that  $AG$  must represent that for the  $W$  pounds of cylinder feed. Through  $A$  and  $G$  it is then possible to draw the water line  $AL$  and the dry-steam line  $GX$  for  $W$  pounds. The cycle for the

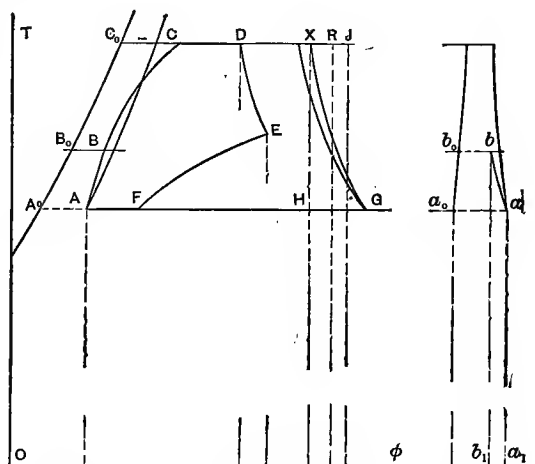


FIG. 97.

Rankine engine using  $W$  pounds per revolution is represented by  $ALXH$ , so that the efficiency of the actual engine as compared with that of the ideal engine is

$$\frac{ABCDEF}{ALXH}$$

In Fig. 98, let  $L'$  and  $L$  be the liquid lines of  $W$  and  $W + w$  pounds respectively, and let  $L''$  be drawn through

the point of compression  $a$  parallel to  $L'$ ; and let  $S$  and  $S'$  be the dry-steam lines corresponding to the water lines  $L$  and  $L''$ . As is evident from Fig. 97, the horizontal distance between the liquid line  $A_oC_o$  and the compression line  $AB$  is equal to the entropy of vaporization of the clearance steam (for example,  $A_oA = a_oa$ ,  $B_oB = b_ob$ ). In Fig. 98 the horizontal distance between  $L'$  and  $L$  represents the entropy for  $W + w - W$  or  $w$

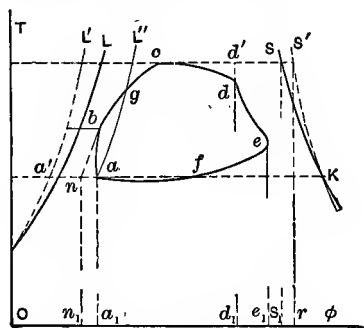


FIG. 98.

pounds of water. Therefore the horizontal distance from  $L'$  to the compression-curve  $ab$  represents the total entropy of the clearance steam.  $a'L'$  is thus the curve of zero entropy for the clearance steam, and any curves, such as  $nb$  and  $aL''$ , parallel to  $a'L'$  represent isentropic changes. The heat rejected along  $ab$  is thus shown, by  $abnn_1a_1$  without the help of any auxiliary diagram.

$a_1 a L'' S' r$  represents the heat received per cycle in the



$W$  pounds of cylinder feed,  $abcdef$  represents the heat utilized, so that the difference,  $M$ , represents the total heat losses. These consist of the exhaust-heat  $a_1aee_1$ , the heat lost during compression  $abnn_1a_1$ , and the heat lost during admission. The heat refunded during expansion is represented by  $dee_1d_1$ . Hence the heat lost during admission equals

$$M - a_1aee_1 - abnn_1a_1 + dee_1d = cL''gcdd_1rS'c - n_1nbgaa_1.$$

If the condition of the steam is desired at any point of the expansion  $de$ , it is found by reference to the lines  $L$  and  $S$  and not with reference to  $L''$  and  $S'$ . Hence the initial condensation changed the state point from  $S$  to  $d'$ , so that this loss is represented by the area under  $Sd'$  and not by that under  $S'd'$ . Subtracting this from the total loss during admission there is left

$$cL''gcdd' + SS_1rS' - n_1nbgaa_1$$

as the heat losses due to wire-drawing and friction.

This method is simple and easy of application, as it requires the construction of only two extra curves,  $L'$  and  $S'$ . Furthermore it gives a very complete account of the clearance steam. The only objection, apparently, is that, with the exception of the expansion line  $de$ , it gives an entirely false idea of the cycle of the cylinder feed, as this in reality is entirely condensed and then heated along  $aL''$  in the boiler.

If the cylinder is jacketed, the heat given out by the jacket steam may be indicated at the right of  $XH$  (Fig. 97), as shown under  $XJ$ . Further, if the radiation loss is known, it may be represented under  $JR$ , and then the remaining area under  $XR$  will represent the heat given by the jackets to the steam in the cylinder.

**Separate Cycles for the Cylinder Feed and the Clearance Steam.**—Whatever the assumptions made with reference to the card, its total area must not be changed. Thus, even if an attempt is made to draw separate cycles for both cylinder feed and clearance steam, the compression line of the resultant cylinder-feed card will never exactly coincide with the water line of the  $T\phi$ -diagram, so that this line must always remain imaginary in its readings. It is, however, possible to have the reconstructed expansion line represent the true  $pv$ -history of the steam. Thus, in place of drawing an isentropic line through the point of compression, draw the polytropic curve  $pv^n = C$ , where  $n$  has the value found for the expansion-curve. Assuming that the clearance steam is expanded and compressed along this line, the card for the cylinder feed can be constructed by assuming this curve to represent zero volume. The  $T\phi$ -projection of the expansion-curve will thus represent the true average history of the cylinder feed, and the compression line will follow more closely the water line than it does when an adiabatic curve is taken as the new base line.

The cycle for the clearance steam can be found between cut-off and release, and between compression and admission, but the rest of it would be entirely imaginary. Possibly it might be continued from admission up to the attainment of initial pressure by assuming adiabatic compression. As may be seen from Fig. 96, the clearance steam contains less moisture at admission and compression than at cut-off and release respectively, so that whatever its exact path between admission and cut-off and between release and compression, it must at least show decreasing and increasing values of  $X$  respectively, so that its history is the reverse of that shown by the indicator-card itself.

It is doubtful if the increased labor involved in the making of such a plot would be recompensed by any added information which could not be obtained by a proper interpretation of the simple method used by Prof. Boulvin.

## CHAPTER XIV.

### STEAM-ENGINE CYLINDER EFFICIENCY.

THE thermal efficiency of an engine, that is, the fractional part of the heat energy received which it changes into useful work, is limited first by the ideal efficiency which is fixed as soon as the initial quality and pressure and the back pressure are known, secondly, by the cylinder efficiency which is a measure of the heat losses in the cylinder, and finally, by the mechanical efficiency which is a measure of the mechanical frictional losses of the whole engine and is equal to the brake horse-power divided by the indicated horse-power. The thermal efficiency is thus the product of these three separate efficiencies, or

$$\eta_{\text{total}} = \eta_{\text{Rankine}} \cdot \eta_{\text{cylinder}} \cdot \eta_{\text{mechanical}}.$$

In order to make the total thermal efficiency a maximum the engineer must so design the engine that each of the component efficiencies possesses its maximum value. The factors influencing the Rankine efficiency have already been discussed in Chapters XI and XII, and this is, in any case, determined by

the running conditions in the plant. The mechanical efficiency is controlled by workmanship and the choice of proper metals for bearing surfaces and of suitable lubricants. It is thus a mechanical and not a thermal problem, and so falls beyond the scope of this treatise. The cylinder efficiency being the result of purely thermal processes can, as shown in the preceding chapter, be submitted to the temperature-entropy analysis.

In order to minimize the heat losses in a steam-engine it becomes necessary to understand the various factors which produce them. The losses are of three kinds:

- (1) Loss of availability due to throttling during admission and exhaust;
- (2) External radiation and conduction loss
- (3) Condensation and re-evaporation inside the cylinder.

The throttling losses depend upon the relative areas of piston and steam ports, as well as upon the speed of the piston and the length and crookedness of the ports. In the case of compound engines the length and cross-sectional areas of the receivers and piping between the cylinders must also be considered. For a given engine the higher the speed the greater the throttling loss.

The external loss depends upon the difference between the mean temperature of the cylinder walls

and the outside air, upon the area of the radiating surface and the conductivity of the materials used. A high-pressure cylinder will radiate more heat per unit area than a low-pressure cylinder; a steam-jacketed cylinder will lose more heat externally than a non-jacketed cylinder, because of its higher temperature and larger surface. This loss is small in comparison with the condensation loss and can be minimized by lagging the cylinder with some suitable insulating material.

The condensation and re-evaporation loss is caused by the tendency of heat to equalize the temperature of bodies in thermal contact. Thus the cylinder and piston are exposed to steam fluctuating in temperature between that corresponding to the pressures at admission and exhaust. Investigations have shown that the fluctuations in temperature of the metal extend only a slight distance, two or three hundredths of an inch, into the mass of the cylinder. It is thus almost a surface phenomenon and the mass of metal participating in it is proportional (practically) to the surfaces exposed to the steam. The quantity of heat transferred is proportional to the difference of temperature between the steam and metal, the area, and the time. Of course if the time interval is long enough the metal will be raised from the temperature of exhaust to that of admission, and then the heat required will equal the weight of metal involved times its specific

heat times the change in temperature, but at higher speeds the temperature of the steam fluctuates so rapidly that there is not sufficient time for the metal to arrive at either the temperature of admission or exhaust, and it thus fluctuates between some intermediate temperatures. Another way of looking at this is to assume that the metal directly in contact with the steam assumes its temperature immediately and then by conduction successive layers are brought to the same temperature. This requires time, so that the higher the speed the thinner the layer of metal which is involved in the operation. From either point of view the only way to eliminate this heat transference would be to run the engine at infinite speed. Comparative tests made upon various engines by setting the governors so that they could run at different speeds with the same conditions of cut-off, and of initial and back pressures always show that the condensation loss is nearly proportional to the time required for a revolution.

Increased speed means, however, increased port area to prevent increased throttling loss and thus increased area of radiating surface with its consequent loss. The number of revolutions is also limited by various mechanical factors, so that the speed in any given case may be considered as fixed. It therefore remains to investigate the effect upon initial condensation of the other elements of design at the disposal of the designer, viz.,

- (1) Size of unit;
- (2) Point of cut-off;
- (3) Compounding;
- (4) Steam jacketing;
- (5) Use of superheated steam.

(1) The mass of steam contained in a cylinder, other things being the same, increases as the cube of the dimensions, while the area exposed to the steam increases only as the square of the dimensions. Thus the radiating surface per unit weight of steam diminishes as the first power with increasing size. That is, doubling the dimensions, or increasing the volume eight times, reduces the initial condensation *per pound of steam* in the larger cylinder to one-half its amount in the smaller cylinder; trebling the diameter and stroke reduces the condensation to one-third its original amount.

(2) The position of cut-off in the high-pressure cylinder controls the weight of steam admitted to the cylinder and therefore the power developed per stroke. The longer the period of admission the greater the horse-power. Does the thermal efficiency or economy increase at the same time, or does it fluctuate, and if so, at what point does it attain its maximum value? Roughly speaking, the cylinder head and the piston and that portion of the cylinder exposed to the steam up to the point of cut-off have their temperatures raised to that of the entering steam, while the tempera-



ture of the rest of the cylinder is raised varying amounts as the temperature of the steam drops during expansion. As cut-off changes the heat required to warm the metal also changes slightly, because the portion of the cylinder heated to admission temperature changes with the cut-off, but a large part of the surface, viz., the cylinder-head, the piston, and the inside of the steam-ports, remains the same, so that although the total variation in cooling surface increases and decreases with the cut-off it is at a much slower rate. The heat required to warm the cylinder per stroke is thus but slightly variable, being nearly the same for wide variations in cut-off, but the weight of steam increases with the cut-off so that the loss of heat per pound diminishes as the cut-off increases. The maximum condensation loss is thus found at early cut-off and the minimum loss when steam is taken through out the entire stroke. Apparently from this point of view the economy of an engine increases with the power developed.

If no heat losses occurred in the cylinder, expansion from early cut-off would carry the pressure at the end of the stroke below back pressure, then as cut-off increased the final pressure would also increase, and at some definite point of cut-off would just equal the back pressure. From this point on the final pressure would exceed the back pressure, and, finally, when cut off occurred at the end of the stroke there would

be no drop of pressure whatever. Due to the condensation during the first part of the expansion the pressure in the actual cylinder drops more rapidly than during adiabatic expansion, so that the cut-off which will give a pressure at the end of expansion just equal to the back pressure is somewhat later in the actual than in the theoretical case. If cut-off occurs later than this the pressure at release drops, without the performance of work, down to the back pressure; that is, the availability of its internal energy is wasted.

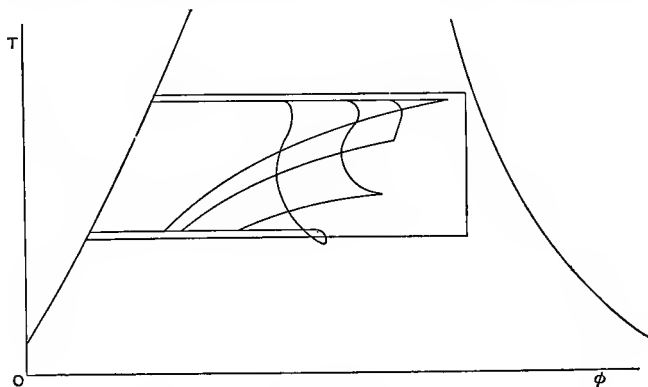


FIG. 99.

Thus the more cut-off exceeds that value at which the expansion would bring the pressure down to exhaust pressure the greater the wasted internal energy. Considered from this aspect alone the engine would first run with a negative loop in the card, so that its economy would increase up to the point at which the loop dis-

appeared, and from this point onwards the economy would decrease slowly as the power increased.

There are thus two factors exerting opposing influences: initial condensation which diminishes, and loss of internal energy which increases with the load. The mutual variations of these factors cannot in the present state of our knowledge be predicted from point to point, but it is at least evident that there is some cut-off at which the sum of these two losses will prove a minimum (Fig. 99). To determine this point of most economical cut-off for any given engine economy tests must be run at varying loads for the same boiler and back pressures. Then the efficiency and cut-off must be plotted as coordinates and the minimum point of the curve thus established will be the desired cut-off.

(3) The heat absorbed by the cylinder metal each revolution is proportional to the change of temperature which the metal undergoes; this in turn depends upon the difference in temperature of the entering and leaving steam. Therefore the greater the range of pressure, or better, of temperature, in a cylinder the greater the initial condensation. Halving the temperature range halves the condensation loss. Thus if an engine is to work between fixed temperature limits the initial condensation loss may be reduced by subdividing this temperature drop among several cylinders, that is, by compounding the engine. The steam condensed in the first cylinder is re-evaporated during

exhaust and it can thus be utilized in the second cylinder to produce power or to heat the walls. The same steam could be used over and over in successive cylinders to heat the walls, so that the condensation losses are not additive. Therefore the greater the number of cylinders the smaller the condensation loss.

But the addition of each new cylinder entails an extra throttling loss in its admission- and exhaust-ports, an extra external radiation loss from its walls and from the piping and receiver which connect it to the preceding cylinder. These radiation and transference losses are practically proportional to the number of cylinders.

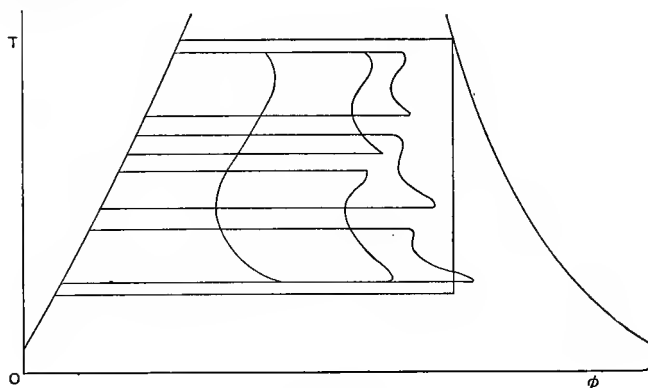


FIG. 100.

In compounding there are thus two opposing factors to be considered: the initial condensation loss which diminishes, and the transference and radiation losses which increase with the number of cylinders. As

before, the theoretical development of the subject proves inadequate and the proper degree of compounding for different pressure ranges must be determined experimentally.

Fig. 100 shows the character of these losses for simple, compound, and triple expansion, between the same initial and final conditions.

(4) Watt stated that the function of a steam-jacket was to heat the engine cylinder to the temperature of the entering steam. If a jacket entirely fulfilled this requirement initial condensation of steam inside a cylinder would be entirely eliminated. This would mean that the cooling of the cylinder walls by low-temperature steam during expansion and exhaust must be made up before admission by heat taken from the jacket. During expansion the steam would receive heat from the hotter walls and its pressure would thus be maintained at a higher value than that corresponding to adiabatic expansion. Some work would thus be performed by the heat received from the jacket. But this heat, thus received at low temperatures, could not be as efficient as the high-temperature heat contained in the entering steam. During exhaust the walls, being at the temperature of live steam, give heat directly to the steam which heat is carried away without performing any work. In fact by increasing the volume of the exhaust steam it may actually increase the throttling loss during exhaust. The steam-jacket by

increasing the external temperatures and dimensions of the cylinder also increases the external radiation and conduction losses.

The steam-jacket is thus not only an inefficient method for supplying heat to the inside of the cylinder for the development of work, but it also increases the external losses. The increased economy obtained by the use of a jacket indicates that the decrease in initial condensation must more than offset the losses inherent in the jacket. The reason for this is readily found. Although steam is ordinarily nearly dry at the throttle the moisture at cut-off may range anywhere from 25 per cent. to 50 per cent. The steam at release usually contains somewhat less moisture than at cut-off, but not to any great amount. The re-evaporation of this moisture during exhaust extracts a large quantity of heat from the metal which must be made up by the entering steam. If now the jacket by preventing initial condensation permits of dry steam at cut-off, a lesser weight of steam would be required and at release this would not contain more moisture per pound than that naturally incident to adiabatic expansion. There would thus be a smaller quantity of water to be evaporated during exhaust, and consequently correspondingly less heat would be required from the cylinder walls. Further, all the heat lost to the cylinder walls through initial condensation and re-evaporation is entirely wasted, and only the

heat of the liquid at *exhaust temperature* can be returned to the boiler, but some of the heat given to the cylinder by the jacket does work during expansion and the heat of the liquid can be restored to the boiler at boiler temperature. Experience has shown that in most cases

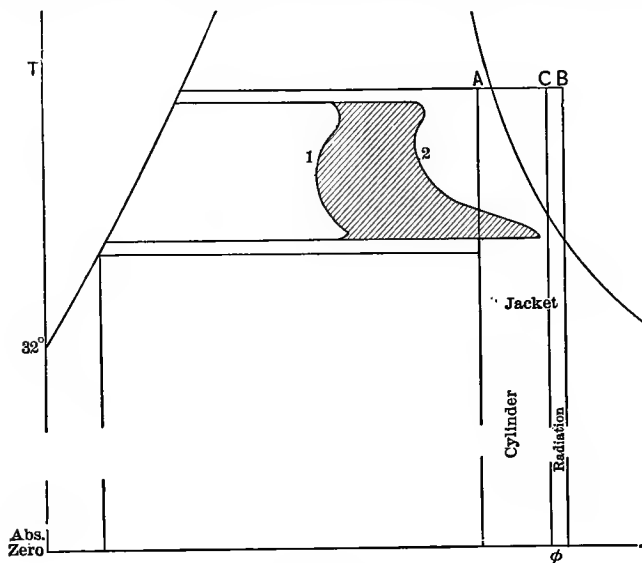


FIG. 101.

a net saving is obtained from the use of a jacket. Naturally, the engine having the largest condensation loss would be most benefited by the addition of a jacket. Thus a jacket would produce a greater saving on a small engine than on a large one, on a slow-speed engine than on a high-speed (speed refers to number of revolutions and not the distance traversed by the

piston in feet per minute), at early cut-off than at late cut-off.

There is of course no expression for ideal efficiency involving jacket steam, as heat conduction losses do not exist in an ideal engine, but in any actual engine the thermal efficiency of the indicated horse-power is given by

$$\begin{aligned}\eta_{\text{I.H.P.}} = \eta_{\text{R.}} \cdot \eta_{\text{cylinder}} &= \frac{2545 \times \text{I.H.P.}}{\text{Steam per hour} \cdot (H_1 - q_2) + \text{jacket steam per hour} \times r_1} \\ &= \frac{2545}{w_c(H_1 - q_2) + w_j \cdot r_1},\end{aligned}$$

where  $w_c$  and  $w_j$  are cylinder feed and jacket steam per I.H.P. per hour respectively.

If the card is projected into the  $T\phi$ -plane everything is established upon the basis of 1 lb. of cylinder feed, so that the expression for the efficiency would be

$$\frac{\text{Area of card in } T\phi\text{-plane}}{H_1 - q_2 + \frac{w_j}{w_c} \cdot r_1}.$$

It is thus possible to indicate the heat received from the jacket steam by laying off a rectangle  $ab$ , Fig. 101, equal to  $\frac{w_j}{w_c} \cdot r_1$ . If the external radiation loss is known this can be further subdivided into  $ac$  and  $cb$ , representing heat given to the cylinder feed and that lost in radiation respectively.



The cylinder efficiency can be obtained from the above by dividing by the Rankine efficiency.

When jackets are used on intermediate and low-pressure cylinders the jacket steam is at a higher temperature than the cylinder feed during admission and can thus more readily perform its function.

The use of jackets on and reheating coils in intermediate receivers has the primary object of drying the exhaust from the preceding cylinder before it passes on to the next. If the steam is not very wet, or if the moisture is removed by a separator, they may serve to superheat the steam. Certain tests seem to indicate that unless they superheat the steam they fail in their purpose.\*

(4) As mentioned on pp. 236-239 the use of superheated steam has but slight effect upon the Rankine efficiency. Thus with steam pressure of 200 lbs. absolute and exhaust at 1 lb. absolute the Rankine efficiency for dry saturated steam is 30.0 per cent., while with steam superheated 300° F. the efficiency is only 31.3 per cent. The large saving actually effected by superheated steam must therefore be due to its effect upon the cylinder efficiency. The explanation is found in the observed fact that heat is conducted less rapidly between a gas and a metal than between a liquid and a metal. Thus Ripper (*Steam Engine Theory and Practice*, p. 149) found in a certain small

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\* Trans. A. S. M. E., vol. xxv, pp. 443-501.

engine "that for each 1 per cent. of wetness at cut-off, 7.5° F. of superheat must be present in the steam on admission to the engine to render the steam dry at

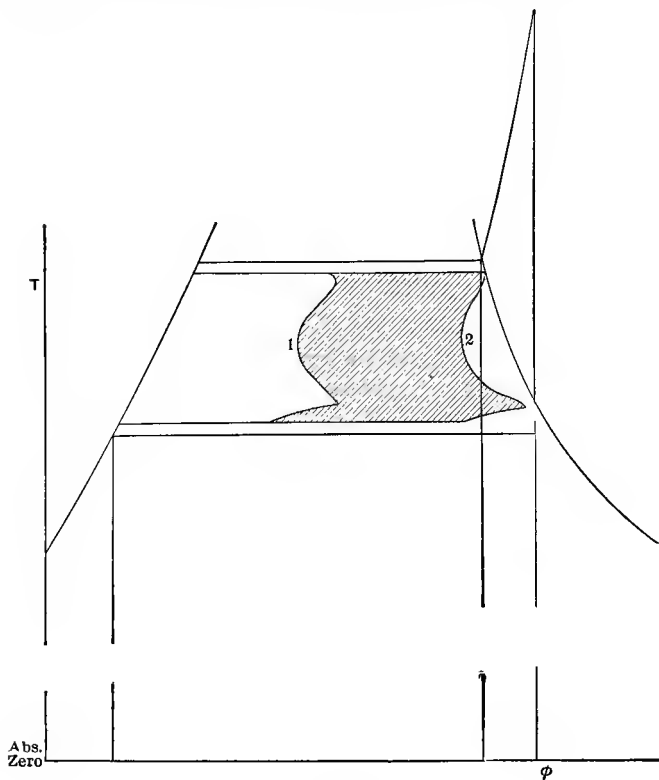


FIG. 102.

cut-off." The loss of a small amount of superheat prevents a larger condensation due to the decreased rate of flow. And furthermore the steam containing less moisture at release, less heat is removed from the

cylinder metal, and consequently less heat is required from the next admission steam.

Let 1, Fig. 102, represent the expansion line when dry-saturated steam is supplied to the engine and 2 when sufficient superheat is present to give dry steam at cut-off. The cross-hatched portion represents the extra heat utilized per pound because of the addition of the small amount of superheat. This represents a case where the heat received per pound,  $H_1 - q_2$ , is increased by about one-fifth or one-sixth, while the heat utilized per pound is nearly doubled.

**Comparison between Reciprocating Engines and Turbines.**—In the steam turbine there is nothing analogous to the initial condensation and re-evaporation losses of the reciprocating engine. After the turbine is once in operation a constant temperature gradient is established from the admission to the exhaust. That is, the steam is continually diminishing in temperature as its pressure drops in the successive nozzles and stages, and the adjacent metal soon acquires the same temperature as the steam. Thus the heat conduction losses are of two kinds: transference of heat through the metal from the steam to the atmosphere and transference of heat through the metal by conduction from steam of high temperature to steam of lower temperature. Both of these losses are probably relatively small as compared with the condensation and re-evaporation losses of the reciprocating type.

Expansion in the reciprocating engine must not be carried beyond the point at which the difference of pressure upon the two faces of the piston becomes less than the frictional resistance of the engine, as such an operation would mean the wasting of power already realized. In other words, part of the heat theoretically available for work must be sacrificed to avoid a greater friction loss. In the turbine, however,

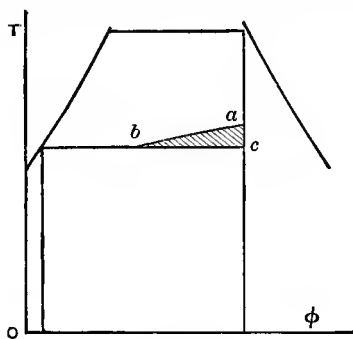


FIG. 103.

no such limitation is placed upon the expansion. In fact the greater the expansion in a turbine the smaller the friction loss, as the major part of the friction is due to windage, and this decreases as the density of the steam surrounding the rotor diminishes. It would thus seem that the turbine has a distinct advantage over the reciprocating type, as complete advantage may be taken of the theoretical possibilities by expanding completely down to back pressure with a net saving of the heat area *abc*, Fig. 103.

In the reciprocating engine the throttling losses during admission and exhaust are comparatively small, and exist primarily because of the impossibility of making the ports large enough consistent with good design to prevent drop in pressure. In the turbine type, on the other hand, wire drawing occurs because of the impossibility of making the clearance spaces about the blades small enough to prevent it. Thus in each stage a considerable portion of the steam passes by the blades without performing work upon them.

Each type of motor thus possesses its own method of dissipating the availability of the heat in the entering steam, and, judging from economy tests, the total losses are about the same in both cases.

## CHAPTER XV.

### LIQUEFACTION OF VAPORS AND GASES.

SUPERHEATED steam of the condition shown at  $a$  in Fig. 104 might be changed to saturated steam by one

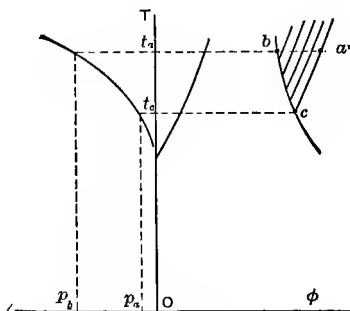


FIG. 104.

of two methods. First, it could be kept in a hot bath at constant temperature and the pressure increased from  $p_a$  to  $p_b$ , so that its state point would move from  $a$  to  $b$ . Secondly, it might be kept under constant pressure  $p_a$  (as a weighted piston) and its temperature allowed to drop by radiation from  $t_a$  to  $t_c$ , so that the state point travels from  $a$  to  $c$ . *Liquefaction will*

*begin by either process as soon as the state point reaches the dry-steam line.*

It is at once evident that the second method is the only one always applicable, for the isothermal change might take place above the critical temperature and then no increase of pressure, however great, could result in liquefaction.

The critical temperature of steam is beyond the upper limit of temperature used in engineering, so that this is not as clear here as in the case of some vapor which superheats at ordinary temperatures, as, for example, carbon dioxide.

Fig. 105\* shows the  $T\phi$ -,  $pv$ -,  $Tp$ -, and  $v\phi$ -diagrams for  $\text{CO}_2$ . The chief differences between this diagram and that for steam lie in the fact that the critical temperature is included, thus showing the intersection of the liquid- and saturated-vapor curves, and further, that the volume of the liquid is now appreciable with reference to that of the vapor and its variation with increasing pressure and temperature no longer negligible.

If  $a$  (Fig. 105) represent the state point of the superheated  $\text{CO}_2$  vapor at the pressure and temperature under consideration, isothermal compression will fail to produce condensation, although cooling at constant pressure will produce liquefaction as soon as  $c$  is reached.

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\* This diagram is only approximately correct, being based upon somewhat discrepant data given by Amagat, Regnault, and Zeuner.

The ordinary gases, hydrogen, oxygen, nitrogen, etc., have their critical points as much below ordinary

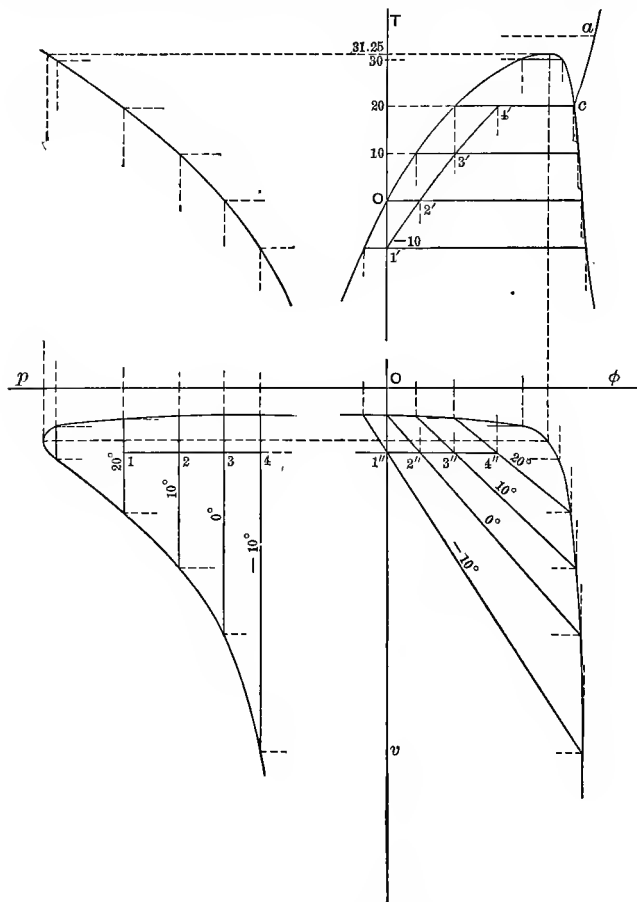


FIG. 105.

engineering temperatures as that of water is above them.



Superheated steam at ordinary temperatures could be liquefied isothermally because its temperature is less than that of the critical point; carbonic dioxide at ordinary atmospheric temperatures could also ordinarily be liquefied isothermally because the usual atmospheric temperature is less than  $31.9^{\circ}\text{C}$ . ( $89.4^{\circ}\text{F}$ .); hydrogen, oxygen, etc., cannot be liquefied isothermally simply because the atmospheric temperature is far above their critical temperatures. The essential factor in liquefaction, then, is to reduce the temperature and then simply to compress the gas isothermally until liquefaction commences.

Suppose it is desired to liquefy some carbon dioxide some summer day when the temperature of the atmosphere is above its critical temperature. Let the compression be carried on slowly, so that the heat generated may be dissipated by radiation and the process be isothermal. Liquefaction will not occur. It will be necessary to cool the gas down to the critical temperature by some means, physical or chemical. Possibly a coil containing cold water will suffice in this case. Proceeding to other substances possessing lower and lower critical temperatures, cooling mixtures giving lower and lower temperature would be required. That is, by this method it would never be possible to liquefy any substance, however great the pressure applied, unless there already existed some source of cold as low as its critical temperature. When the "permanent"

gases are reached the sources of artificial cold fail, and unless the gas may be made to cool itself investigation must cease. Any new gas thus liquefied of course in turn becomes a new source of cold to aid in further investigation.

Let Fig. 106 represent the  $T\phi$ -diagram of some gas having a low critical temperature. Consider the

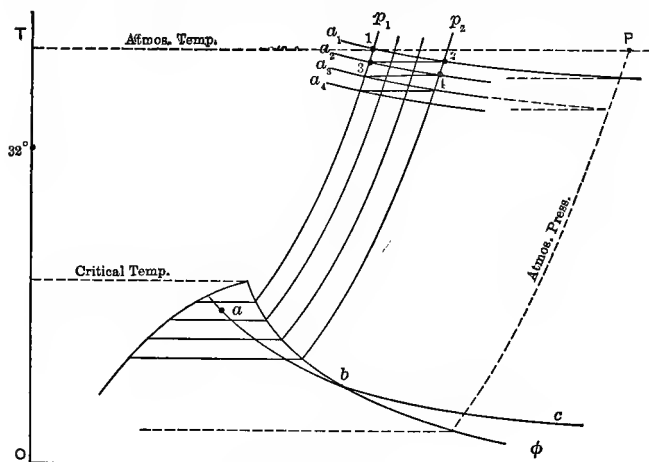


FIG. 106.

throttling-curve  $abc$ . By definition this represents an adiabatic change, during which no work is performed; i.e., the heat contained in the substance is a constant. For this curve the first law of thermodynamics gives

$$A\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = 0 = A(E_1 - E_2 + p_1v_1 - p_2v_2),$$

which becomes

$$E_1 - E_2 = p_2v_2 - p_1v_1;$$

that is, the curve represents an irreversible isodynamic process in the case of a perfect gas and differs but slightly from it for ordinary gases.

Now, the internal energy of any substance is defined as the summation of the sensible heat and the disgregation work, or the kinetic energy of the molecules due to their own vibrations plus the potential energy due to their mutual positions. Representing these quantities by  $S$  and  $I$  respectively, it follows that  $S_1 + I_1 - S_2 - I_2 = p_2 v_2 - p_1 v_1$ .

The work necessary to separate the molecules against their mutual attraction must increase with the distance between them although the rate of increase is inversely as the square of the distance between them.

As the volume increases the value of  $I$  increases and hence the value of  $S$  must decrease an amount equal to

$$I_2 - I_1 + p_2 v_2 - p_1 v_1;$$

that is, the temperature of the substance decreases.\* With increasing volume the rate of temperature drop decreases so that the curve  $abc$  approaches  $T = \text{const.}$  as an asymptote. Near the saturation curve, in the region of the "superheated" vapors, this drop is considerable, but far to the right of this curve and above the critical temperatures these "throttling" curves become almost parallel to the  $\phi$ -axis. The "perfect

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\* For hydrogen at ordinary temperatures  $p_2 v_2 - p_1 v_1$  is negative and greater than  $I_2 - I_1$ , so that the temperature increases.

gas" is simply the limiting condition in which the potential energy has attained its maximum value, or rather where any limited change in volume does not affect the total value of  $I$  appreciably.

In such a case

$$p_2 v_2 = p_1 v_1,$$

$$E_1 = E_2,$$

and

$$S_1 + I_\infty = S_2 + I_\infty;$$

that is, the disgregation change being negligible, the isodynamic curves become coincident with the isothermals.

In Fig. 106 let  $a_1, a_2, a_3$ , etc., represent a series of such throttling-curves. Let a pound of air be taken from its initial condition  $P$  (representing atmospheric pressure and temperature) and be compressed isothermally to 1. This may be effected by jacketing the cylinder walls of the compressor with cold water. If the air is then permitted to expand along the throttling-curve  $a_1$ , the temperature will drop, say, from 1 to 2. The air at reduced pressure is fed back to some intermediate stage of the compressor. In some forms of liquefiers the expansion is carried at once down to atmospheric pressure, thus securing a somewhat greater drop in temperature, and the air is then returned to the first stage of the compressor. If this cooled, expanded air be made to flow back outside the pipe containing more air from the compressor, this in turn will be

cooled by conduction to some lower temperature and so will escape from the throttling-valve at pressure  $p_1$  at some lower temperature (3). This will now expand along the throttling-curve  $\alpha_2$  to a still lower temperature (4). This process is continued until the temperature of the issuing jet has fallen below the critical temperature, when liquefaction will ensue.

This will evidently occur first in the nozzle, as the temperature outside will need to be still further decreased before the air will remain liquid at the reduced pressure. The vaporization of the liquid first formed tends to decrease still further the temperature of the air-tubes, etc. It is probable that at first all of this liquid vaporizes as soon as ejected, but the vaporization of part soon cools down the rest and its surroundings, so that a small portion remains liquid at the lower pressure. The back pressure may thus in time be reduced to that at  $P$  and then the temperature will be found at which air vaporizes at atmospheric pressure.

The expansion of the air thus provides in itself the cooling process needed to reduce the temperature below the critical point so that sufficient increase of pressure may cause liquefaction.

## CHAPTER XVI.

### APPLICATION OF THE TEMPERATURE-ENTROPY DIAGRAM TO AIR-COMPRESSORS AND AIR-MOTORS.

**The Reversed, Power-absorbing, Thermodynamic Processes.**—In the direct power producing thermodynamic processes already considered, whether the simple theoretically perfect Carnot cycle, or the various practical cycles such as the Rankine cycle for saturated and superheated vapors, or the Ericsson and Stirling cycles for air, or the Otto, Joule or Diesel cycles for internal-combustion engines, the fundamental principle of operation is the same; namely, during the expansion of the working fluid heat is absorbed at highest possible temperatures, and during the compressive stroke heat is rejected at the lowest possible temperatures, the difference between the amount of heat absorbed and the heat rejected being transformed into mechanical energy.

In all of these processes the amount of work produced by means of the cycle is always much less than the energy supplied to the working substance, because a necessary condition for the transference of heat into mechanical work is a temperature difference. The greater this temperature difference the greater the

fractional part of the heat which can be transferred into work. This temperature difference is therefore limited by the range in temperature between that at which the heat is supplied and that of the coldest of the surrounding bodies, which ordinarily is that of the atmosphere or of the cooling water.

In all these cycles the maximum efficiency seldom ranges above 30 per cent, more ordinarily 25 per cent, but under exceptional conditions the Carnot efficiency may reach 50 per cent. In the actual engine working approximately on any of these cycles the fractional part of the heat received which is changed into work is naturally considerably less than the fractional part theoretically available, due to heat losses of varying kinds and magnitude. In other words, we may sum up the properties of these cycles as follows:

Heat is taken in at high temperature; a portion of this heat is changed into work, but most of it is rejected as heat at lower temperatures.

In the reversed power absorbing thermodynamic cycles the working fluid is made to traverse its thermodynamic history in a reverse direction; namely, it takes in heat at low temperatures while expanding, is then compressed and gives out heat at high temperatures, the power required to perform this compression being equal to the power which in the direct cycle is produced by the expansion of the working fluid.

In the usual definition for efficiency the efficiency

is stated as output divided by input. For the direct cycles this magnitude is always less than unity, but for the indirect cycles its value, theoretically at least, is always greater than unity, although in the actual working mechanism heat and friction losses may serve to reduce it to practically unity, and sometimes even less.

As we analyze this reversed cycle we find that the output may be of two different kinds, i.e., the cycle may be utilized to remove heat at low temperatures from some confined space (as for example the ordinary refrigerating machinery), or may be utilized to deliver heat at high temperatures to some confined space, as in the ordinary heating and ventilating systems. In both of these cases the input represents the work required to drive the compressor, the output, however, corresponds in the first case to the exhaust heat of the direct cycle, and in the second case to the heat received in the direct cycle. Thus, for example, in the use of steam, the efficiency of the Rankine cycle under ordinary conditions is approximately 20 per cent. Therefore the exhaust heat represents 80 per cent of the total heat available for supply to the engine. In other words, the exhaust heat is four times the magnitude of the utilized heat. Therefore in the reversed cycle, if we consider the cooling effect as useful output, it is evident that this cooling effect is four times the magnitude of the force required to drive the compressor,



while if we consider the heating effect of the exhaust steam at high temperature, it would have five times the magnitude of the power required to drive the compressor. Again the suction of the compressor might be used for refrigeration and its discharge for heating, and in such a case the effective thermal output or useful effort would be nine times that required to drive the compressor.

A third use of the compressor is the production of gas under high tension for future use in power production, physical experimentation, and industrial use.

**The Compressor.**—In all of these systems the compressor plays an essential part, and it is therefore well to make a study of the compressor cycle first, before entering into the details of the different cycles embodying this reversed process. The compressor is really an engine driven backward, with all of its parts, valve gears, etc., operating in a reverse order. The usual cycle of operation is something as follows:

Consider the piston to be at the beginning of the stroke; a certain small quantity of air is compressed in the clearance space under the pressure prevailing in the delivery pipe. As the piston moves forward this clearance air expands behind it until the pressure has decreased slightly below that in the suction pipe. The small difference of pressure then prevailing between the suction pipe and the inside of the cylinder suffices to lift the suction valve off its seat against the spring

which usually holds it. As the piston continues to move forward new air will rush in behind it to the end of the stroke. It is quite possible that the inertia of the air in the suction pipe will be such that even after the piston slows down at the end of the stroke the air will continue to rush into the cylinder, and may even bring the pressure in the cylinder up to that prevailing in the suction pipe. The admission valve now closes under the action of the spring. As the piston starts

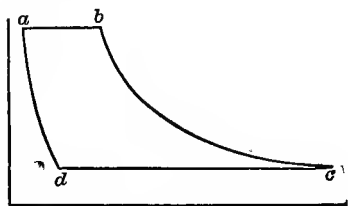


FIG. 107.

to retrace its motion it crowds the air ahead of it into the further end of the cylinder, so that the pressure continually rises until it reaches such a magnitude as to slightly exceed that in the delivery pipe; when the small difference of pressure thus created suffices to lift the discharge valve from its seat, and then during the rest of the stroke the air will be delivered into the discharge pipe. This cycle repeats itself indefinitely. Referring to Fig. 107, the cycle is as follows: From *a* to *d* the gas (or vapor) in the clearance space expands until the pressure at *d* falls sufficiently below that in the supply pipe to permit the admission valve to open

and admit a new supply from *d* to *c*. On the return stroke, the entire quantity is compressed along *cb*, until the pressure becomes sufficient to lift the release valve, when discharge occurs from *b* to *a* against the upper pressure. The indicator-card thus shows the entire cycle of the clearance gas, but only one portion, the compression, of that of the charge. The expansion and compression of a gas in a cylinder is more nearly adiabatic than that of a saturated vapor, and hence, as the temperature of the gas at the end of admission is nearly that existing at the end of expansion, the expansion and compression of the gas in the clearance space may be considered to neutralize each other thermodynamically; although mechanically the greater the clearance the greater the size of the cylinder necessary to compress a given amount of gas. Therefore all discussion of the effects of clearance will be omitted for the present.

During the expansion of the clearance air and the compression of the total charge, the heat interchanges between air and cylinder walls are much smaller than the corresponding changes between the steam and cylinder walls in steam engines, i.e., the operations are nearly adiabatic. If atmospheric air were simply a mixture of oxygen and nitrogen the equation of such a compression or expansion would be  $pv^{1.405} = C$ , but due to the fact that a certain amount of moisture is always present in the air the value of the exponent

is somewhat different. Thus diatomic gases, as  $N_2$ ,  $H_2$ ,  $O_2$ , etc., possess a ratio of specific heats

$$\frac{c_p}{c_v} = 1.4, \text{ approximately,}$$

but as the complexity of the molecular composition increases the value of this ratio decreases. Thus for superheated  $H_2O$  the value is approximately 1.31 to 1.33, so that the mixture of atmospheric air and superheated water vapor gives a value of this ratio which is something less than 1.4. It is of course evident that this theoretical value is not actually realized in the air compressor cycle, because certain heat interchanges must inevitably take place, so that in place of adiabatic compression the compression line shows slightly decreasing pressures as the volume decreases, i.e., decreasing below the values which you would expect from adiabatic compression. Nevertheless, the deviation from the true adiabatic compression is so small that most engineers consider any marked deviation to be indicative of leakage rather than of thermal interchanges.

In case the compressor forms part of a heating engine the increase in the temperature during compression is an essential feature, and every effort should be made to prevent loss of temperature through external radiation and conduction, but if the temperature at which the heat is delivered is of no importance and the total value of

the operation consists either in removing heat for refrigerative purposes or in storing up compressed fluids for future power purposes, the work required for adiabatic compression represents a real loss, because the same result could theoretically be accomplished by isothermal compression to the upper pressure limit. Thus, in the case of air-compressors supplying air for air-motors such as rock-drills, etc., the hot air delivered by the compressor must, during its transmission through the pipe-line, be reduced in temperature to that of the surrounding atmosphere and therefore the air occupies, at the pipe-line pressure, that volume which it would attain provided it underwent isothermal compression from the initial condition, and therefore the difference between the work of adiabatic compression and the work of isothermal compression represents useless expenditure of power.

In order to obtain in a motor power equal to that required for compression, it would be necessary to pre-heat the air up to the final temperature of compression before it entered the motor. Actually without pre-heating the expansion of the air in the motor cools the temperature of the exhaust to such a point that the water entrained in the atmosphere frequently freezes in the exhaust ports.

In Fig. 108, let  $ab$  and  $ac$  represent isothermal and frictionless adiabatic compression respectively from some lower pressure  $p_1$  to a higher pressure  $p_2$ . If the

temperature of the cooling water is the same as that of the atmosphere, the minimum possible expenditure of work in compressing from  $a$  to  $b$  equals that under the isothermal  $ab$ , or  $p_a v_a \log_e \frac{v_a}{v_b}$ . If, however, the

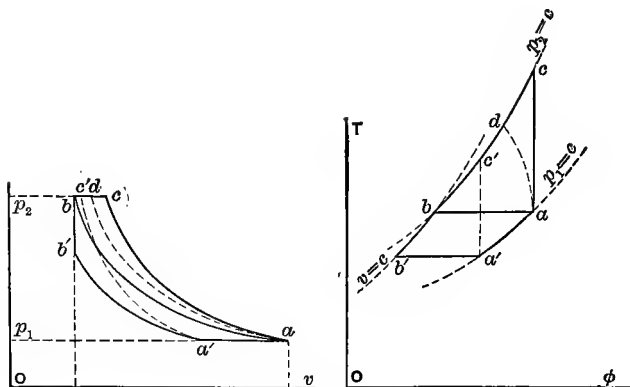


FIG. 108.

compression is adiabatic it will be necessary for the delivered air to contract at constant pressure, losing by conduction and radiation the heat  $c_p(t_c - t_b)$ . That

is, the work  $\frac{p_a v_a}{k-1} \left[ \left( \frac{p_c}{p_a} \right)^{\frac{k-1}{k}} - 1 \right]$  is performed upon the gas during compression, as shown under  $ac$ , and the work  $p_b(v_c - v_b)$  during contraction in the storage tubes or coil, as shown by  $cb$  in the  $p$ - $v$ -diagram. The wasted work is thus shown by the area  $abc$ , and has the value

$$\begin{aligned}
& \frac{p_a v_a}{k-1} \left[ \left( \frac{p_c}{p_a} \right)^{\frac{k-1}{k}} - 1 \right] + p_b (v_c - v_b) - p_a v_a \log_e \frac{p_b}{p_a} \\
&= \frac{p_a v_a}{k-1} \left[ \left( \frac{p_c}{p_a} \right)^{\frac{k-1}{k}} - 1 \right] + p_a v_a \left[ \left( \frac{p_c}{p_a} \right)^{\frac{k-1}{k}} - 1 \right] \\
&\qquad\qquad\qquad - p_a v_a \log_e \frac{p_b}{p_a} \\
&= \frac{k}{k-1} \cdot R T_a \left[ \frac{T_c}{T_a} - 1 \right] - A T_a [\phi_a - \phi_b] \\
&= A c_p [T_c - T_a] - A T_a [\phi_a - \phi_b].
\end{aligned}$$

In the  $T\phi$ -diagram the minimum amount of heat rejected is shown under  $ab$ , while that rejected during contraction at constant pressure after adiabatic compression is shown under  $cb$ , the heat wasted by the latter process being shown by  $acb$ . If, as is usually the case, the compression line lies somewhere between these two extremes, the wasted work and heat will be represented by some such area as  $adb$ .

If the cooling water is colder than the atmosphere, it is, theoretically at least, possible to reduce the necessary work by cooling the entering gas at constant pressure from  $a$  to  $a'$ , compressing isothermally to  $b'$ , and then permitting the gas to warm up at constant volume by taking heat from the atmosphere. The work performed and the heat rejected during compression are represented by the areas under  $a'b'$  in the two diagrams, and the work saved by  $aa'b'b$ . If the compression from  $a'$  is along the adiabatic  $a'c'$ , the saving over that along  $ac$  is shown by the area  $aa'c'c$ .

It is not possible to cool the hot gas very much by

jacketing, but the waste work may be reduced by dividing the compression into two or more stages and cooling the gas in intermediate coolers to the initial temperature.

Thus suppose the compression to follow the law  $p_1 v_1^n = p_2 v_2^n$ , and to be represented by the curve  $ac$  in Fig. 109. Instead of completing the compression in one cylinder, stop at some intermediate pressure  $p_x$ , at  $d$ ,

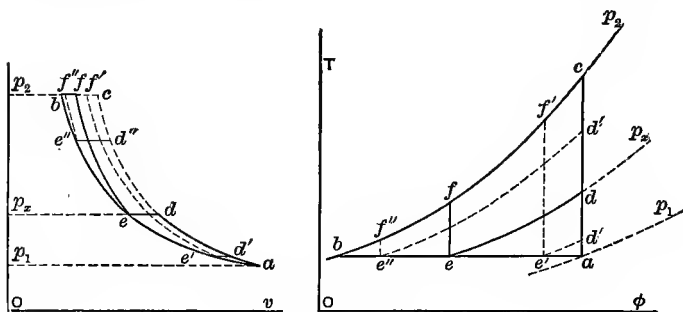


FIG. 109.

and cool under constant pressure to  $e$ . Continue the compression in a second cylinder along  $ef$ , and finally cool at constant pressure along  $fb$ . The wasted work or the heat ejected is no longer represented by the whole of  $abc$ , but by the two portions  $ade$  and  $efb$ ; that is, the work or heat saved by compounding is represented by the area  $cdef$ . From the diagram it is at once evident that this area approaches zero as  $p_x$  approaches either  $p_1$  or  $p_2$ , and that there is some intermediate position which gives the maximum saving. The



proper value of  $p_x$  is easily found from the expression for work

$$W = W_{ade} + W_{efb} = \frac{n}{n-1} \cdot p_a v_a \left[ \left( \frac{p_x}{p_1} \right)^{\frac{n-1}{n}} + \left( \frac{p_2}{p_x} \right)^{\frac{n-1}{n}} - 2 \right].$$

This expression has its minimum value when

$$\frac{d}{dp_x} \left[ \left( \frac{p_x}{p_1} \right)^{\frac{n-1}{n}} + \left( \frac{p_2}{p_x} \right)^{\frac{n-1}{n}} \right] = 0,$$

i.e., when  $p_1 : p_x = p_x : p_2$ , or  $p_x = \sqrt{p_1 p_2}$ .

Fig. 110 shows similar diagrams for a three-stage compressor with intercoolers. The saving thus introduced is shown by the irregular-shaped figure  $defghc$ . This again varies in magnitude with the values of

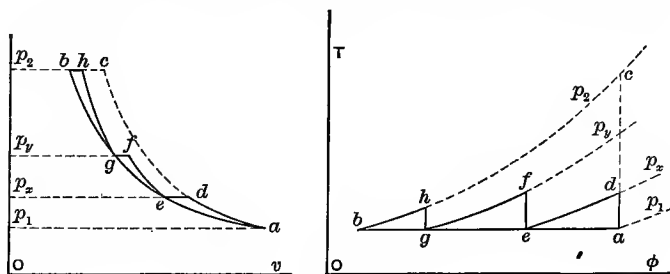


FIG. 110.

$p_x$  and  $p_y$ , and in a manner similar to the above may be shown to have its maximum value when

$$p_1 : p_x = p_x : p_y = p_y : p_2,$$

or  $p_x = \sqrt[3]{p_1^2 p_2}$  and  $p_y = \sqrt[3]{p_1 p_2^2}$ .

Naturally the greater part of the saving is obtained by the use of one extra cylinder, the gain from successive extra cylinders becoming correspondingly smaller.

**Taylor Hydraulic Air-compressor.**—To make the compression line coincide with the isothermal throughout, would require an indefinite number of stages, but as the introduction of each new stage increases the total friction loss of the compressor the number of stages actually feasible is limited to three, or possibly four. There is, however, one type of compressor essentially simple in construction which realizes almost perfectly this ideal condition. This is what is known as the Taylor hydraulic air-compressor, consisting mainly of a well, sunk at some point where a drop of water is available, as at any waterfall, and containing a submerged bell-shaped chamber which is connected through the top by a pipe-line leading to the upper level of the water, this pipe-line extending downward in the bell part way from the top. Water from the upper level flows through this pipe into the bell, keeping the lower end of the pipe submerged.

To introduce the compressive feature there is a series of small tubes with the lower ends inserted in the water entering the top of this tube. The rush of the water serves to draw bubbles of air down through these tubes, which bubbles are caught in the descending water and carried downward into the bell. Here the stream is deflected by horizontal aprons, thus giving

the air a chance to separate from the water by gravity; the air collecting in the top of the bell, the water passing out underneath the lower edge of the bell up around the entire mechanism to the lower level of the fall. By this means the air in the top of the bell is subjected to a pressure equal to the weight of water corresponding to the head, measured from the top of the water in the submerged bell up to the lower level of the water at the fall. It is evident that the energy input of this compressor is equal to the weight of water passing through the tube, multiplied by the height of the water-fall. The useful output, on the other hand, is represented by the work necessary to compress the actual quantity of air delivered isothermally from atmospheric pressure to the pressure prevailing in the submerged bell. Evidently to obtain a maximum efficiency the in-rush of air should be so regulated that the maximum quantity of air possible should be carried down by the water. The actual loss in this instrument is occasioned by the buoyancy effect due to gravity. The air particles as they pass down through the tube are subjected to ever greater pressures, decrease in volume, and thereby assume greater density, so that the buoyancy effect diminishes as they descend in the tube. The buoyancy effect, however, tends to make the bubbles move upward relatively to the falling water, so that part of the work already accomplished in moving them downward is wasted by this retrograde action.

**The Influence of the Clearance Space.**—Returning once more to the piston compressor it is next necessary to consider the effect of the clearance space. As we have seen, the air under high pressure, stored in the clearance space at the beginning of the stroke, expands as the piston moves outward, and fills a certain portion of the piston displacement, thereby decreasing the space actually available for new air during the suction stroke. In other words, the actual effective displacement of a compressor is less than its apparent displacement, as determined from its dimensions and length of stroke. The ratio of the actual displacement to the apparent displacement is known as the displacement efficiency of the compressor. Sometimes the actual air delivered per cycle divided by the apparent displacement is also called displacement efficiency, but as this ratio also includes the effect of leakage by the piston and valves its use should be avoided. It is better to have separate ratios: one defining the influence of the clearance, the other the effect of leakage upon the delivered air. Therefore, in calculating the size of the compressor to perform a stated amount of work so as to deliver a definite volume of free air, the piston displacement calculated directly from the volume of the air to be compressed and delivered must be divided by this displacement efficiency in order to obtain the actual piston displacement. The power required to drive the actual compressor (neglecting friction losses) is

the same as the power that would be required to drive a compressor having no clearance space, because, as the heat interchange is small between air and cylinder walls, the expansion and compression lines are practically the same, so that the extra work required to compress this clearance air is returned by expansion of the air during the suction stroke.

If  $x$  = the increase in volume of the clearance air during expansion from delivery ( $p_2$ ) to suction ( $p_1$ ) pressure, and  $n$  = the exponent of the expansion curve, while Cl. and P.D. represent the volume of the clearance and piston displacement respectively, there exists the simple relation

$$p_2 \cdot \text{Cl.}^n = p_1 (\text{Cl.} + x)^n$$

whence 
$$x = \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \cdot \text{Cl.}$$

From this it follows that the actual displacement equals

$$\begin{aligned} & \text{P.D.} - x \\ & = \text{P.D.} - \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \cdot \text{Cl.} \end{aligned}$$

The displacement efficiency thus becomes equal to

$$1 - \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right] \frac{\text{Cl.}}{\text{P.D.}}$$

**Determination of the Clearance of an Air-compressor from the Indicator Card.**—(a) It may be assumed without much error that the values of the exponent

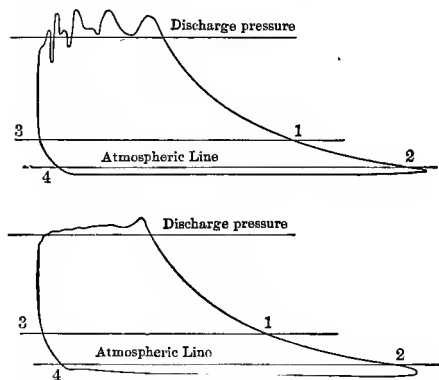


FIG. 111.

$n$  for the expansion and compression curves are the same. Let these curves be intersected by any two straight lines parallel to the atmospheric line in the points 3 and 4 and 1 and 2, respectively (in Fig. 111 the atmospheric line is utilized), so that  $p_3 = p_1$  and  $p_4 = p_2$ . Determine  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  in per cent. of the piston displacement, and let  $c$  represent the volume of the unknown clearance in per cent. of the piston displacement.

Then

$$p_1(c + v_1)^n = p_2(c + v_2)^n \quad \text{and} \quad p_3(c + v_3)^n = p_4(c + v_4)^n,$$

whence

$$\frac{c+v_1}{c+v_3} = \frac{c+v_2}{c+v_4},$$

or

$$c = \frac{v_2v_3 - v_1v_4}{v_1 + v_4 - v_2 - v_3}.$$

As neither  $p$  nor  $n$  enter into the value of  $c$ , neither the scale of the indicator spring nor the actual law of expansion and compression need to be known.

As the percentage errors are much larger in the determination of  $v_3$  and  $v_4$  than in  $v_1$  and  $v_2$ , the accuracy of the method may be increased by assuming a probable value for the clearance and adding this to  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$ . The resulting value of  $c$  thus obtained will be not the clearance itself, but the difference between the actual and assumed value, and may be either positive or negative according to the error in the assumption.

(b) When, as is usually the case, the scale of the card is known greater accuracy may be attained by using only the compression line. Intersect the curve at  $p_1$  and  $p_2 = \frac{1}{2}p_1$ , and also at  $p_3$  and  $p_4 = \frac{1}{2}p_3$ . Determine the corresponding volumes  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$ . Then

$$p_1(c+v_1)^n = p_2(c+v_2)^n,$$

or

$$2 = \left( \frac{c+v_2}{c+v_1} \right)^n,$$

and

$$p_3(c+v_3)^n = p_4(c+v_4)^n,$$

$$\text{or} \quad 2 = \left( \frac{c + v_4}{c + v_3} \right)^n,$$

$$\text{whence} \quad c = \frac{v_1 v_4 - v_2 v_3}{v_2 + v_3 - v_1 - v_4}.$$

**Actual Air-compressor Indicator Cards.**—Fig. 111 shows indicator cards from the two cylinders of a small single-stage air-compressor in the laboratories of the Institute. In the lower card the discharge-valve was functioning properly, in the upper card it opened and closed spasmodically. The admission-ports of the lower card offered more resistance than those in the upper card, as shown by the dropping suction line.

The indicator card does not represent a closed cycle, and hence if projected into the  $T\phi$ -plane only portions of it have any significance, namely, the compression and expansion lines and the enclosed area. It is, however, instructive to study the compression line, as this gives some clue to the effectiveness of the jacketing. In any case, the compression line should not deviate much from the adiabatic. The presence of a large deviation is indicative of a leak instead of a heat interchange.

**Various Efficiencies.**—As we have seen, part of the work of compression is, as far as useful output is concerned, wasted energy, so that it has become customary to consider the power required for isothermal compression of the air to represent the useful output of the



compressor and to define the ratio of this isothermal work to the actual indicated work as the air efficiency, or sometimes as compressor efficiency. In the case of the ideal engine the air efficiency would equal the ratio of the isothermal work to the adiabatic work for the proper number of stages.

Usually such compressors are driven either by direct connected motors or steam engines, or are belted to such motors. There is usually no attempt made to separate the friction losses in such combined units of the motor and of the compressor, but the total friction loss is obtained by determining the difference in the indicated power of the steam cylinders and the air cylinders, and the mechanical efficiency of such a unit is defined as the ratio of the indicated air power to the indicated steam power. Of course in the case of an electric motor operating such a compressor the efficiency of the unit would be represented by the ratio of the indicated air power to the electrical input at the terminals of the motor.

The values of these various efficiencies actually attained in practice vary between very wide limits due to different forms of compressors and the varying degrees of workmanship. Thus the displacement efficiency ranges between 78 per cent and 94 per cent, i.e., the effect of the clearance is to decrease piston displacement by 22 to 6 per cent of its actual value.

The air efficiency or the compression efficiency is

influenced by the action of the admission and delivery ports and also by the heat interchanges. The heat losses are greater in compound than simple compressors because the air comes in contact with two cylinders instead of one. In certain of the better types of compressors the valves are not operated by difference of pressure, but by mechanical means. In these latter compressors the throttling loss during admission and exhaust is much diminished. Numerically the efficiency of compression varies from about 50 to 90 per cent. On page 371, Peabody, there is a table giving the values found in tests on a variety of compressors.

The mechanical efficiency of the unit is ordinarily in the neighborhood of 85 per cent, but occasionally reaches higher values as, for example, page 372, Peabody, mechanical efficiency for blowing engines at Creusot is quoted at 92 per cent. From the values given for the air and mechanical efficiencies it is evident that the total efficiency of the compressor unit, which may be defined as the isothermal work of compression divided by the indicated power of the steam cylinders, ranges from 40 to 75 per cent.

It is interesting to compare these values with those obtained with hydraulic compressors. Thus reports of tests made upon the Taylor Hydraulic Compressor show that the ratio of the isothermal work on the air to the water work ranges from 60 to 70 per cent. This corresponds to the total efficiency of a steam-

driven air-compressor unit and shows that the results obtained for Taylor compressors always exceed the minimum values of the steam-driven compressor, and usually the values very nearly equal to the maximum one obtained with steam-driven compressors. It is of course possible to use water power to operate a water turbine and to connect this turbine to a piston air-compressor. Such water turbines range in efficiency from 20 to 80 per cent. If for a basis of comparison we take this maximum efficiency of the water turbine as 80 per cent and multiply it by a high value for the efficiency of compression (say 80 per cent) this would give as the total efficiency of a water turbine air-compressor unit a value of 64 per cent, which is readily exceeded by the Taylor compressor.

**Applications of Compressed Air.**—Compressed air is used in almost all branches of mechanical and chemical engineering, the scope of this utilization being indicated briefly by the following headings: (1) Mechanical Draft, Cooling Towers, Ash and Coal Conveyors, (2) Heating and Ventilating, (3) Air Drying, Lumber Drying Kilns, etc., (4) Steel Production and Manipulation, as Forges, Hammers, Drills, Riveters, etc., (5) Air Motors and Rock Drills, (6) Exhausters to Promote Increased Speed of Evaporation, etc., (7) Pneumatic Conveying Systems, Planing Mill Exhausters, (8) Sand Blast for Cleaning Iron, Stone, etc., (9) Paint Blast for Painting Bridges, Freight Cars, etc., (10) Pumps of both Displacement and

Expansion type, of which latter the Pohlé-air lift is a good example, and the combination of both methods, such as the Starrett air pump.

**Types of Compressors.**—These various applications call for air pressures varying from a fraction of an ounce to the square inch up to possibly ten pounds to the square inch. In the case of the air-motors and for the production of liquid air, and some few other devices, much higher pressures extending up to the hundred or thousand, or several thousands, of pounds to the square inch are required. To produce air of such varying pressures a variety of mechanisms has been devised. For low pressures extending up to one ounce or less per inch, a disk or propeller fan running at slow speed will handle large volumes of air at small cost. For pressures up to one pound per square inch centrifugal fan blowers consisting of steel wheels running at high speeds in conical casings will handle large volumes of air at comparatively small cost. In both these types of fans there is always the possibility for air to leak back from the delivery side to the suction side through the clearance space, so that for higher pressures some more positive action or mechanism must be utilized. Thus for pressures ranging to ten pounds we have a rotary type of blower or exhaustor engine which consists essentially of two geared wheels so intermeshing that on the upward stroke or motion air is carried forward between the teeth, but on the downward stroke the teeth intermeshing prevent the escape

of the air, so that it is delivered into the discharge pipe. These blowers handle capacities ranging anywhere from five cubic feet to over 15,000 cubic feet per minute

The piston compressor is designed to deliver air at any pressure up to three or four thousand pounds per square inch. The Taylor hydraulic compressor occupies a unique position in that it attains most closely to isothermal compression. It is capable of supplying large volumes of air at ordinary commercial pressures.

**Use of Compressed Air for Pumping.**—As a simple, economical, and reliable power for pumping purposes, compressed air for many applications cannot be surpassed. The ordinary types of air pumps fall under two general headings, (1) those using the pressure of the air alone to displace water from a suitable chamber, (2) those using the expansive force of the air by intermingling it with the column of ascending water, as in the Pohlé air lift. There is a third type which is really a combination of these two, a good illustration being the Starrett pump.

In the displacement type of pumps the compressed air is led into the top of the pump chamber and the water is forced out through a pipe from the bottom. When the water is completely displaced the supply of air is shut off and the air in the chamber is either exhausted to the atmosphere or in some types is fed back during the suction stroke of the compressor. The weight of water in the delivery pipe closes the check valve and

prevents water flowing back into the chamber. The decrease in pressure in the chamber, however, permits water to enter through an admission pipe either under the influence of gravity (in case the chamber is submerged) or under the influence of atmospheric pressure in case the air pressure is reduced by drawing it into a compressor.

Ordinarily such pumps are made with two cylinders, so that one is discharging while the other is filling, thus giving a fairly constant delivery of the water. Such a method of pumping utilizes simply the direct pressure of the air and fails to utilize any of its expansive force or internal energy. The economy obtained by such a method must be comparable with that obtained in direct-acting steam pumps where the steam is used non-expansively.

**The Pohlé Air Lift** consists of two tubes, one a large tube with a bell-shaped enlargement at the bottom, submerged to a considerable extent in the well from which water is to be pumped. The other is a small pipe for supplying the compressed air which it delivers underneath the bell-shaped opening of the larger tube. The result is that the compressed air rising upward under the influence of gravity tends to carry with it slugs of water. We have thus in this tube a rising column of alternating layers of water and air, or a column of water interspersed with numerous air bubbles. This results in a diminished density of the combined mass, so that the weight

of water outside the pipe will balance a considerably higher head of this mixture inside the tube. The greater the height to which the water must be raised the less the density of the mixture in the tube. It is to be noticed that this Pohlé air lift is simply the reverse of the Taylor air-compressor. Thus, as the bubbles rise in the pipe, they are subjected to ever decreasing pressure, namely, the weight of the water above them, and therefore expand. The tendency to cool, due to expansion, is offset by the rapid interchange of heat from the water to the air, due to the intimate mixture. The expansion of the air is therefore practically isothermal, and the work developed is thus the maximum which could possibly be attained without the use of an external source of heat. The sources of loss in this pump are (1), in the absorption of air by water so that the whole of the air supplied cannot be utilized for pumping purposes, and (2), in the fact that water from the bottom of one slug tends to leak downward to the top of the one following, so that there is a continual backward flow of water down the pipe, i.e., water once lifted falls back and must be pumped up again, thereby wasting part of the power of the air. Experiments on certain Pohlé air lifts have shown an efficiency of about 50 per cent.

**The Starrett Air Pump** utilizes a combination of the displacement pump and the Pohlé air lift. Essentially it consists of two displacement chambers connected to a common discharge pipe and supplied through an auto-

matic valve from a common air pipe. One cylinder is filling under the influence of gravity, while the other is being emptied by means of the compressed air. By means of a rather complicated valve action a portion of the air used for displacement escapes into the column of ascending water, the rest of it being discharged directly to the air. The valve also bleeds a certain quantity of compressed air directly into the discharge pipe. It is evident that all the air discharged to the atmosphere escapes with its internal energy unutilized, and therefore fails to give as good economy as the air which passes into the delivery pipe. Theoretically, at least, such a pump cannot have as high a thermal efficiency as the simple Pohlé air lift. It has, however, certain other advantages which in many installations would make it more advantageous than the Pohlé air lift. Thus the Pohlé air lift requires that the pipe be submerged to such a distance beneath the level of the water that the weight of the water outside the tube shall overbalance the longer column of water and air inside the tube. In other words, such a method could not be utilized unless the well was sunk far below the level of the water. The Starrett pump, on the other hand, may be placed just below the surface of the water, as the water flows into it under the influence of gravity, and therefore does not need such a deep excavation. It is quite possible that the decreased cost of installation might offset any thermal advantages inherent in the Pohlé air lift. Tests



made upon the Starrett pump installed in the laboratories of the Institute have shown efficiencies of about fifty per cent, which for some strange reason seem to be higher than the efficiencies which are quoted for the Pohlé air lift.

There is one interesting feature common to both the Pohlé and the Starrett pump, and that is that during the practically isothermal expansion of the air in the delivery pipe heat is flowing from the water to the air, therefore increasing the work developed by the air; or, in other words, part of the heat energy of the water is being utilized to help pump it.

**Compressed Air Used as a Source of Power.**—Returning to the first general case where the air is used for power, it is necessary first of all to discuss the influence of the pipe line which conducts the air from the compressor to the machine to be operated by the compressed air, such as a rock-drill, a penumatic riveter, a compressed-air motor, etc. The temperature of the pipe may be considered as equal to that of the surrounding atmosphere and hence constant. The heat generated by friction is thus at once dissipated by conduction, and there thus results an isothermal drop of pressure and increase of volume.

Thus, if *abcde*, Fig. 112, represent the passage of the air through the compressor, *ef* shows the loss experienced by the air in flowing from the compressor to the engine. Suppose the air at *f* to expand adiabatically in the motor

down to back pressure, the amount of work performed will be equal to the area under  $fg$  in the  $pv$ -diagram. The exhaust air is now warmed to the initial temperature along the constant-pressure curve  $ga$ , thus per-

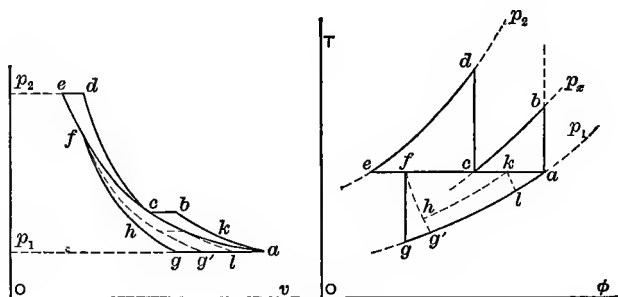


FIG. 112.

forming upon the atmosphere the work under  $ga$  in the  $pv$ -plane, and receives from the atmosphere the heat under  $ga$  in the  $T\phi$ -plane. The maximum amount of work could be obtained from such an engine if the expansion were along the isothermal  $fa$ . This can be partially attained by jacketing with water at atmospheric temperature, so that the actual expansion-curve lies somewhere between these two limiting cases, as at  $fg'$ . A further gain could be made by compounding the engine and heating the air up to atmospheric temperature in the intermediate receiver, as indicated by  $fhkla$ .

**Effect of Pre-heating the Air.**—The amount of work theoretically obtainable from a pound of air in a non-conducting motor is given by the formula:

$$\begin{aligned}
W &= \frac{k}{k-1} [p_1 v_1 - p_2 v_2] = \frac{c_p}{A} \cdot T_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \right] \\
&= 778 \times 0.2375 T_1 \left[ 1 - \frac{14.7^{\frac{0.405}{1.405}}}{p_1^{0.2883}} \right] \\
&= 184.8 T_1 \left[ 1 - \frac{2.17}{p_1^{0.2883}} \right] \\
&= \left[ 184.8 - \frac{401.0}{p_1^{0.2883}} \right] \cdot T_1,
\end{aligned}$$

and is thus shown to be directly proportional to its initial temperature. In the operation of motors by compressed air it is thus possible to bring about a considerable saving in the amount of air required as well as an improvement in the thermal efficiency in any given case by heating the air.

This heating, or pre-heating, as it is generally termed, may be accomplished either by:

1. The direct application of heat in some form of combustion heater, or
2. The admixture of steam.

This latter method may operatively be effected by one of two methods, viz., by blowing steam into the air main or by passing the air through a boiler, in which latter operation it becomes heated to the temperature of the steam.

**Air-motor Economy.**—In air-motor work it is customary to quote the consumption in terms of cubic feet of free air per indicated horse-power per hour. This result may be derived from the preceding equation as follows:

$$\begin{aligned}\text{Lbs. of air per I.H.P. hour} &= \frac{33000 \times 60}{184.8 \left[ 1 - \frac{2.17}{p_1^{0.2883}} \right] \cdot T_1} \\ &= \frac{1072}{\left[ 1 - \frac{2.17}{p_1^{0.2883}} \right] \cdot T_1}\end{aligned}$$

Cubic feet free air per I.H.P. hour = lbs. air  $\times$  vol. of 1 lb. air at atmospheric temperature

$$\begin{aligned}&= \frac{1072}{\left[ 1 - \frac{2.17}{p_1^{0.2883}} \right] \cdot T_1} \times 12.39 \times \frac{T_a}{491.5} \\ &= \frac{270.2 T_a}{\left[ 1 - \frac{2.17}{p_1^{0.2883}} \right] \cdot T_1}\end{aligned}$$

The maximum air consumption evidently occurs when the air in the pipe-line has been cooled to atmospheric temperature, and is equal to

$$\frac{270.2}{1 - \frac{2.17}{p_1^{0.2883}}} \text{ cu.ft. per I.H.P. hour.}$$

The cubic feet for any amount of pre-heating may then be obtained by multiplying this result by  $\frac{T_a}{T_1}$ .

**Air-motor Efficiencies.**—Assuming the air to be supplied by a single-stage compressor, and that before entering the motor it is cooled to atmospheric temper-

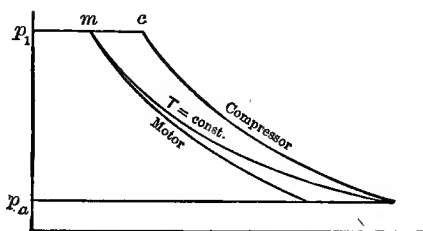


FIG. 113.

ature, then the work of adiabatic compression would be (Fig. 113):

$$\left(184.8 - \frac{401.0}{p_1^{0.2883}}\right) \cdot T_c,$$

so that the efficiency of the compressor-motor unit would be:

$$\frac{T_m}{T_c} = \left(\frac{p_c}{p_m}\right)^{\frac{1-k}{k}} = \eta_{c.m.}$$

If the compressor is driven by a steam engine, the efficiency of the motor is equal to the combined efficiencies of boiler, Rankine cycle, cylinder, mechanical efficiency of engine, compressor unit, efficiency of motor compressor, or

thermal efficiency of motor =

$$\eta_{\text{boiler}} \cdot \eta_{\text{Rankine}} \cdot \eta_{\text{cylinder}} \cdot \eta_{\text{mechanical}} \cdot \eta_{\text{compressor}} \cdot \eta_{\text{motor}}.$$

The value of this thermal efficiency lies well inside ten per cent.

**Influence of Pressure and Temperature upon the Indicated Work.**—The expression for the work developed in an air motor:

$$W_m = \frac{c_p}{A} \cdot T_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \right],$$

and the corresponding expression for the work required to compress air:

$$W_c = \frac{c_p}{A} \cdot T_2 \left[ \left( \frac{p_1}{p_2} \right)^{\frac{k-1}{k}} - 1 \right]$$

may be transformed the one into the other by making use of the expression for an adiabatic expansion in terms of temperature and pressure:

$$T_1 p_1^{\frac{1-k}{k}} = T_2 p_2^{\frac{1-k}{k}}.$$

These must of course be of the same magnitude, as they represent the same cycle of operations simply in a reversed direction. In both formulæ the subscript 1 applies to the conditions at the upper pressure level, and subscript 2 the conditions at the lower pressure level. These different forms for the same quantity are

used because ordinarily one knows the initial condition of the air supplied to a motor and would not be liable to know the exhaust temperature, and similarly, one knows the condition of the air supplied to the compressor and would not be liable to know its exhaust temperature.

A comparison of these two formulæ makes evident the fact that the amount of work produced or consumed is absolutely independent of the actual values of the upper and lower pressures, but is dependent solely upon the ratio of these values. Thus the amount of work required to compress a pound of air from 1 lb. to 15 lbs. pressure is equal to that which would be required to compress it, say from 15 lbs. to 225 lbs. Similarly the work required to compress a pound of air from .01 of a pound per sq. in. to one pound per sq. in. is as great as the work required to compress it from, say 10 lbs. per sq. in. to 1000 lbs. per sq. in. On the other hand the amount of work obtained or absorbed per pound of air is directly proportional to the initial temperature of the charge supplied to the motor or to the compressor.

If therefore it is essential and necessary to water-cool the cylinders of an ordinary air-compressor which is compressing from atmospheric pressure to two or three hundred pounds, it is just as necessary to cool the cylinders of the dry air pump, whose function it is to take the low pressure air from a vacuum chamber and discharge it into the atmosphere. This is clearly demonstrated in

the temperature-entropy plane from the fact that the constant pressure curves all possess the same form and are simply displaced from one another parallel to the entropy axis, the displacements being proportional to the logarithms of the pressure. Thus the pressure curves for 1000, 100, 10, 1,  $1/10$ ,  $1/100$  lbs., etc., would form a

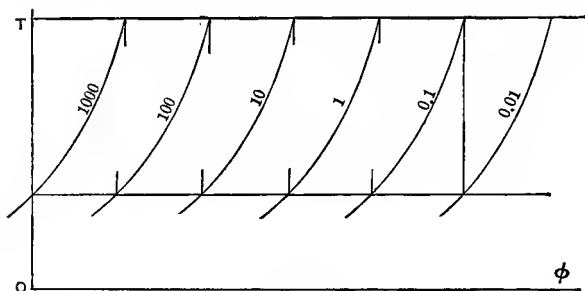


FIG. 114.

series of curves separated by equal distances (Fig. 114). Therefore the work of compression from any one curve to the succeeding higher curve would be the same as that from any other curve to its succeeding curve; assuming the compression in all cases to start at the same temperature, the increase in temperature would be the same in all cases, and finally the heat absorbed by the cooling water must also be the same in all cases.



## CHAPTER XVII.

### DISCUSSION OF REFRIGERATING PROCESSES.

**Refrigerative Units.**—Various branches of engineering have their own units, which in a sense are historical records of the functions formerly exercised by other devices or systems, and which have been usurped by these particular branches of engineering. Thus in power development the name of the horse is preserved in horsepower, *Pferdstaerke*, *cheval vapeur*, etc., which is the unit for all steam- and gas-engine measurements; even in boiler rating the name still sticks, although its numerical value has long ceased to possess any relation with its origin.

In refrigeration engineering it is not the horse but natural ice which is being replaced. As heat conduction is a continuous operation, the refrigerating agent must operate twenty-four hours a day, so that when we fill our refrigerators with ice the supply must be sufficient to last the time interval from one filling to the next. In popular or commercial parlance the refrigerating needs of any plant would be represented by the rate of melting of the ice, as so many units of weight of ice per day. It is but natural, then, to find the capacity of mechanical refrigerating plants expressed as the equivalent of so

much ice melted daily. The unit of refrigeration adopted by the A.S.M.E. is the heat absorbing capacity due to the melting of one ton of ice in twenty-four hours. As the latent heat of fusion of ice is 144 B.T.U. per lb. this unit corresponds to the absorption of 288,000 B.T.U. per 24 hours, or 12,000 B.T.U. per hour, or 200 B.T.U. per minute. A report of the committee to the society may be found in the records of the A.S.M.E. in Vol. 28, year 1906, page 1249.

The output of a refrigerating plant is expressed in various ways, according to the feature it is desired to accentuate. Thus to the purchaser or user, the chief item of interest is the ice equivalent, so the output is quoted in tons of ice melted in twenty-four hours. For the power engineer the important question is the horse-power required to produce a certain number of units of refrigeration, so that the output is usually quoted in B.T.U. refrigeration per steam horse-power per minute. Finally, from the standpoint of cost, reference must eventually be made to the coal pile, so that the refrigerative effect is also quoted in terms of pounds of ice melted per pound of coal burned.

**Refrigerative Systems.**—Refrigerating plants group naturally under one of three headings:

1. Those using air with either the natural or dense air system.
2. Those using some volatile substance, as ammonia, with a compressor,

3. Those using some volatile substance, as ammonia, with some absorbent, as water, with its necessary adjuncts, absorber, generator, analyser, rectifier, inter-changer.

**Air Refrigeration.**—Air refrigeration at the present time is used chiefly under such circumstances as make the use of ammonia refrigeration impossible or inexpedient, the advantage of this method being that leakage of the refrigerative fluid causes no inconvenience. The disadvantage lies in the small heat capacity of the air, which is only 0.2375 B.T.U. per pound per degree difference of temperature, so that large volumes of air have to be handled by the compressor and the rest of the system in order to affect any appreciable refrigeration. The apparatus used consists of several members, each possessing the peculiarities and performing the functions described in the following. A compressor which may if desirable be two-staged, the function of which is to increase the pressure of the air a considerable amount. The involved increase of temperature must next be eliminated by carrying the delivery pipe of the compressor through a suitable water bath, so that the air is restored to atmospheric temperature. The only way in which the temperature of the air can be diminished other than by direct radiation to some colder body, is to utilize the internal energy of the air in the performance of work. This is most effectively accomplished by expanding the air in a motor or expan-

der which ordinarily is either direct connected to the compressor piston rod or indirectly through a common shaft and thus serves to help drive the compressor. The difference between the power absorbed by the compressor and that developed by the motor must be supplied from some external source. Because of the practically adiabatic expansion in the motor cylinder the final temperature of the air is expressed by the formula

$$T_2 = T_{\text{atm.}} \times \left( \frac{p_{\text{atm.}}}{p_2} \right)^{\frac{1-k}{k}},$$

i.e., the air during expansion undergoes the same fractional diminution in temperature as its increase during compression. The air leaving the compressor is therefore much colder than the atmosphere and may be used as a refrigerative agent. The rest of the system consists of a pipe conducting the cold air from the exhaust of the motor to the refrigerating rooms or brine tank, etc. As the air circulates through these various chambers heat flows into it from the surrounding hotter atmosphere and it therefore serves to cool these respective chambers. Eventually the air will regain atmospheric temperature, so that the theoretical refrigerative effect is equal to  $0.2375 \times (T_{\text{atm.}} - T_2)$ . Actually, however, the full value of this refrigerative effect cannot be realized as the latter portion of the heat is received at temperatures greater than that of the desired refrigeration, so that this would be transferred through the piping

after the air left the refrigerative chamber and is on its return to the compressor. Ordinarily, therefore, this final section of the pipe is omitted and the air is wasted, a new supply being drawn in at the compressor. In air refrigerating plants the work performed during expansion is no longer the main object, as in the case just

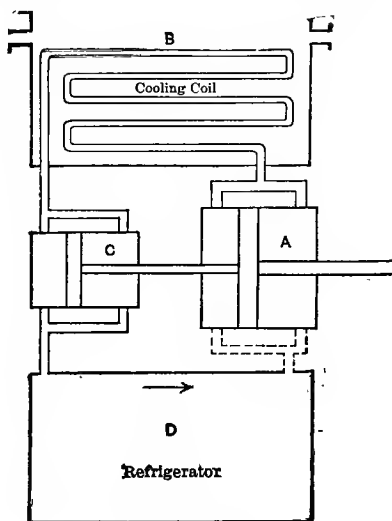


FIG. 115.

discussed, but is simply a means of obtaining the desired end, viz., a sufficient drop in the temperature of the air. The essential parts of such a system are shown in Fig. 115. The compressor *A* takes its supply of air from the atmosphere and discharges the compressed air into the cooling coil *B*, where temperature and volume are both decreased at constant pressure. The cold air now

passes into  $C$ , and in expanding helps to operate the compressor. The expanded air is delivered at low temperature and atmospheric pressure to the refrigerator room, and as it passes through  $D$  its temperature increases and reaches that of  $D$  by the time it leaves at the right. The cycle of the air is completed by warming to the initial atmospheric temperature outside of the

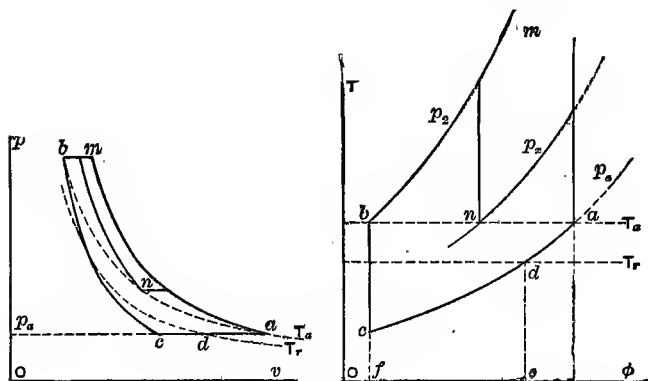


FIG. 116.

refrigerator. The expansion in  $C$  being used to attain low temperature instead of work, care is taken not to heat the air during expansion, so that the expansion may be as nearly adiabatic as possible.

In Fig. 116 let  $anb$  represent the passage of the air through a two-stage compressor  $A$  and the cooling tank  $B$ . During the expansion  $bc$ , in the working cylinder  $C$ , the temperature of the air drops below that maintained in the refrigerator  $T_r$ , the air being delivered at pressure

$p_a$ . As the temperature of the air increases along the constant-pressure curve  $ca$ , it extracts from the refrigerator the heat under the curve  $cd$ , in the  $T\phi$ -plane. The heat under  $da$ , necessary to complete the cycle, is obtained from the atmosphere. That is, the refrigerating effect  $cdef$  is attained by the expenditure of the work  $abcda$ . If the compressor has but one stage,  $amb$ , the efficiency will be correspondingly less.

When we realize that a pound of air at ordinary atmospheric conditions occupies about 13 cu.ft. and for a range of temperature of, say 100 degrees, has a refrigerative capacity of approximately 24 B.T.U., we see that to accomplish any appreciable amount of refrigeration a large number of pounds of air must be circulated and therefore the volume of air to be handled is practically prohibitive. It is at this point that advantage is taken of the fact previously mentioned that the work of compression is independent of the actual pressures, and only dependent upon the ratio of pressures in what is known as the Allen Dense Air Machine. In this mechanism the air is never expanded down to atmospheric pressure, but reaches, as it exhausts from the motor, a pressure of about five atmospheres, or roughly 60 lbs. gage, and when discharged from the compressor is ordinarily about  $15\frac{1}{2}$  atmospheres, or roughly 210 lbs. gage.

Assuming that the air entering the compressor is practically of atmospheric temperature, we see that the volume of one pound has been reduced from approx-

imately 13 cu.ft. to approximately 2.6 cu.ft. In other words, the volume of every part of the system is reduced to about one-fifth of that which would be needed if atmospheric air had been used. The temperatures ordinarily attain, with the Allen Dense Air Machine, from 70° to 90° F. below zero. This system is mostly used on board ship, and the air is used to a certain extent in a regenerative manner, i.e., the coldest air just leaving the motor passes through the ice-making box, and then somewhat warmer next enters the meat chamber, and upon leaving this is still cold enough to be used in chilling the drinking water butt. Finally, as it passes on its way to the compressor, it is used to chill the water-cooled air about to enter the expander.

The amount of work theoretically is the same in the dense air as in the natural air system, but because of the reduced volume the compressor and motor friction losses may be smaller. The cost is less, and this, combined with the decreased floor space, makes it feasible to utilize such a system. Such plants are still used in places where it is more essential to guard against danger arising from the leakage of fluids, such as ammonia, than to install the most economical plant, as, for example, on war vessels.

**Ammonia Refrigerating Plant.**—A complete cycle of the working fluid in the second class of refrigerating plants, where a saturated vapor is used, differs materially from the above, as the substance is condensed



and vaporized during the process. The more commonly used fluids are ammonia, carbon dioxide, and sulphur dioxide; ammonia being used most generally.

Fig. 117 shows diagrammatically the essential features of an ammonia refrigerating plant. It consists of (1) a

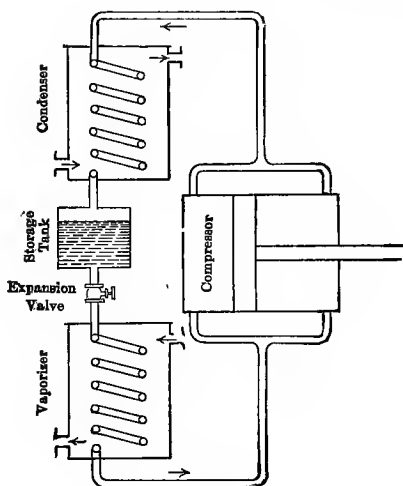


FIG. 117.

compressor which takes the low-pressure vapor from the refrigerator coils and delivers it at some higher pressure, (2) a condenser consisting of a series of coils in which the hot gas is cooled until it liquefies, (3) a storage-tank containing a supply of ammonia which remains liquefied under the high pressure at atmospheric temperature, (4) an expansion-valve from which the liquid emerges under reduced pressure, and (5) the

refrigerator coils in which the liquid, under reduced pressure, is vaporized by withdrawing the necessary heat from its surroundings. The high pressure prevails from the delivery-valve of the compressor to the expansion-valve, and low pressure from the lower side of the reducing-valve to the admission-valves of the compressor.

The refrigerator coils may be used directly, thus bringing the temperature of the surroundings down, near the boiling temperature of the liquid, or, if such a low temperature is not desired, the coils may pass through a bath, as of brine, and reduce this to the desired temperature. The cold brine is then circulated through the refrigerator. This latter method gives a more nearly constant temperature. The least movement of the expansion-valve causes variations in the back pressure, and hence in the boiling temperature of the ammonia, which would affect the surrounding air if used directly, but which would be absorbed by the large heat capacity of the brine. The direct system is, however, simpler and less expensive to install and to maintain.

As long as any liquid remains unvaporized in the refrigerator coils these will remain at the temperature of vaporization, but afterwards the pipes assume the higher temperature of the bath or surrounding atmosphere, and then begin to superheat the vapor at constant pressure. This superheating is still further in-

creased in the pipe leading back to the compressor. After leaving the compressor the highly superheated vapor passes to the condenser, losing part of its superheat at constant pressure on the way. This process is finished in the condenser, and then the vapor begins to liquefy at the temperature corresponding to the high pressure. The liquid finally emerges cooled to the temperature of the cooling water and collects in the storage-tank above the expansion-valve ready for a new cycle.

The corresponding  $pv$ - and  $T\phi$ -cycles are shown in

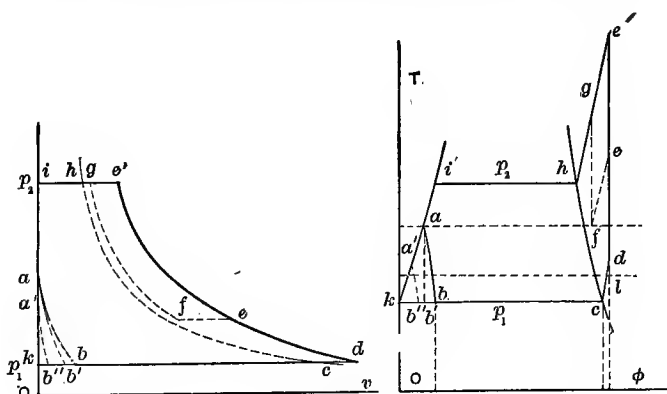


FIG. 118.

Fig. 118,  $ab$  represents the passage of the liquid through the expansion-valve along a constant-heat curve, during which a portion of the liquid  $\frac{kb}{kc}$  is vaporized and the refrigerative power of its liquefaction destroyed;

*bc* represents the vaporization of the remainder of the liquid; *cd*, the superheating of the vapor during the last part of the refrigerator coils and the return pipe to the compressor; *de'* represents the compression, and *e'h*, the loss of superheat by conduction, radiation, etc., as the hot gas flows along the pipe to the condenser. If the compressor were two-stage, with intermediate cooling down to atmospheric temperature, the path followed would be *defgh*. The liquefaction is represented by *hi*, and the further cooling of the liquid down to atmospheric temperature by *ia*.

To decrease the work required to compress the gas, attempts are made in various types of compressors to cool it during compression by the use of water-jackets or by the direct injection into the cylinder of either liquid ammonia or oil. In such cases the compression is no longer adiabatic, but of the form  $pv^n = p_1v_1^n$ , where  $n$  will depend in any given case upon the amount of heat extracted by the jackets or absorbed by the injected fluid.

The temperature of the entering vapor is usually considerably lower than that of the cooling water, so that heat is radiated only during the latter part of the stroke, when the temperature of the vapor greatly exceeds that of the water. The compression line of indicator-cards from such compressors should therefore approximate closely the adiabatic curve drawn through the commencement of the compression stroke

and should begin to fall below it more and more only as the discharge pressure is approached.

If oil of the same temperature as the entering vapor is injected into the cylinder, it can affect the temperature only by absorbing heat as the gas is compressed. The specific heat of the oil is greater than that of the cylinder walls, and possibly conduction occurs somewhat more rapidly from vapor to oil and then oil to metal than it would directly from vapor to metal, especially if the oil is in a finely divided state. This can only result in changing slightly the exponent  $n$  of the compression curve  $pv^n = p_1v_1^n$ .

The effect of injecting liquid ammonia is difficult to describe in general terms, as the results will differ according to the quantity injected and the various temperatures of the liquid, vapor, and cylinder walls. If the cylinder walls are assumed to be non-conducting and the injected liquid is previously cooled to the temperature of the refrigerator, the superheated vapor will then lose its superheat and in so doing suffer a drop in pressure, as the cylinder volume is momentarily constant. At the same time, however, the injected liquid will be partially vaporized, increased in volume, and thus effect an increase in pressure. The resultant effect in this case would undoubtedly be a net reduction of pressure and thus decrease the work of compression. As, however, the cylinder walls are good conductors when in contact with a liquid, enough

of the ammonia might thus be vaporized to produce a net increase in pressure. If the liquid is injected at the same temperature as the entering vapors, it is at a higher temperature than that of saturated vapor at the prevailing pressure, and hence will partially vaporize until a condition of equilibrium is established. What the final conditions of pressure and temperature will be will evidently depend upon the relative weights of liquid and vapor, the pressure, and the temperatures of vapor, liquid, and cylinder. In either of the above assumptions, if the resultant pressure is less and part of the liquid still remains unevaporated, the work of compression would be still further decreased, as saturated vapors transmit heat to the cylinder walls more rapidly than superheated vapors.

If the liquid is injected into the suction-pipe of the compressor, it will expand at the prevailing back pressure and reduce the temperature down to that corresponding to that pressure. Whether or not there results a net diminution in volume must depend upon the amount injected and the quantity of heat received from external sources. Although the work may or may not be decreased, according to circumstances, the temperature will at least be decreased, and thus the amount of necessary cooling water diminished.

It is theoretically possible to effect a further saving in liquid refrigerating plants by changing from the throttling-curve *ab* (Fig. 118) to a frictionless adiabatic

expansion  $ab'$ ; that is, by replacing the expansion-valve with an auxiliary cylinder and thus utilize the expansive force of the ammonia to help run the compressor. The refrigerative power of the ammonia would be increased at the same time, since the amount vaporized during expansion would be decreased from  $kb$  to  $kb'$ . It is possible that the mechanical complications thus introduced would more than counter-balance the thermodynamic savings.

It has also been suggested that the loss in refrigerative power occasioned by the expansion  $ab$  could be diminished by reducing the temperature of the liquid at the point  $a$ . Thus the gas in passing from the refrigerator to the compressor absorbs from the atmosphere the heat represented by the area under  $ld$ . If such a loss is unavoidable, it could be neutralized by jacketing the return pipe with the liquid ammonia about to be fed to the refrigerator, thus reducing the temperature of the latter from  $a$  to  $a_1$  and so decreasing the amount vaporized by expansion from  $kb$  to  $kb''$ .

**Conditions for Maximum Efficiency.**—The ammonia, after leaving the expansion coil, must be as cold as the lowest temperature desired in the refrigerating plant, and after condensation in the cooling coils cannot have a temperature less than that of the cooling water. Evidently the minimum temperature range is covered by these two factors, so that the minimum amount of com-

pression work will be obtained when the ammonia is cooled just to the minimum temperature and is compressed to a pressure corresponding to the temperature of the cooling water. Such conditions necessitate the ammonia entering the compressor as soon as it is completely vaporized. If we call  $T_1$  the upper temperature and  $T_2$  the lower temperature, the refrigerative effect per pound of ammonia will be represented by  $H_2 - q_1$  and the work of compression will be represented by  $H_1 - H_2$ , so that the maximum efficiency under such conditions will be represented by the expression

$$\frac{H_2 - q_1}{H_1 - H_2}$$

From this expression it is evident that the efficiency increases as  $H_1 - H_2$  diminishes. This can be accomplished in two ways: either by raising the lower pressure or diminishing the upper pressure. On the other hand, a decrease in the lower pressure or an increase in the upper pressure will cause a decrease in efficiency. It is evident that decreasing the back pressure makes it possible for the refrigerant to attain the temperature of the refrigerating room only when in a superheated condition. This gives a small increase in the refrigerating effect but a larger increase in the specific volume of the ammonia, so that less ammonia is taken into the compressor per stroke and the compressor must be speeded up to produce the same cooling effect as before



the change. If, on the other hand, the lower pressure is left unchanged and the initial pressure increased, the refrigerating effect per pound remains constant, and the work of compression is increased so that more power would be required to drive the compressor.

In actual practice we fail to realize the ideal efficiency in refrigerators because, (1) all of the heat  $H_2 - q_1$  is not absorbed from the brine or the refrigerating room, but partially from the atmosphere, so that (2) the temperature of the ammonia entering the compressor is hotter than that of the returning brine, or than the refrigerating rooms, thus increasing both  $H_2$  and  $H_1$ , and therefore increasing the difference  $H_1 - H_2$ ; (3) to obtain fairly rapid flow of heat from the ammonia to the cooling water the discharge pressure must exceed the theoretical minimum, thus increasing the work of compression.

The amount of refrigeration of any given temperature decrease is limited only by the speed at which the ammonia compressor can be run. The efficiency of the operation may, however, be impaired somewhat, due to the greater throttling action of the ports and valves.

To obtain refrigeration of greater intensity the throttle valve must be throttled still more. This will result in a reduction of efficiency, since not only is the refrigeration decreased but the work of compression is increased.

**Absorption Refrigeration.**—There are two ways in which ammonia may be returned from the lower to the higher pressure level. One method we have already

discussed; namely, that of compressing it in a suitable compressor and then condensing by means of cold water. The other method consists in using an absorbent, as, for example, water, which is used in all ordinary operations and which possesses the power of absorbing many times its own volume of ammonia gas, and then using a small pump to force the concentrated solution from the lower pressure to the upper pressure and deliver it to a boiler or generator, where the ammonia is liberated from the water by the application of heat. According to the temperature at which the absorption takes place, a pound of water will absorb from a fraction of a pound to something more than two pounds of ammonia. This, expressed in gaseous volume, shows a decrease of two or three hundred volumes to one, i.e., the action of water is equivalent to a perpetual vacuum. In the generator the application of heat evaporates not only the ammonia but also a considerable amount of water vapor, which must necessarily be eliminated before the ammonia can be used as a refrigerating agent. On the top of the generator is a tower-like structure consisting of a series of troughs and passage-ways so designed as to return any condensate to the boiler without hindering the upward passage of the vapors. By the radiation of heat and by the inrush of the cold concentrated ammonia solution from the absorber, the ascending column of vapors is chilled and the less volatile steam suffers the greater condensation. At the same time the incoming

solution is heated and a portion of the more volatile ammonia is driven off and passes out at the top of the tower, together with the ammonia which has come from the generator. Even now the gas is mixed with a certain small quantity of steam which must be eliminated. This is accomplished in a cooling coil known as the "rectifier" and so drained that any condensate flows back into the tower or analyzer. This rectifier is usually placed at some point exposed to the atmosphere, such as the roof of a building, and is cooled by a small quantity of water trickling down over the pipes, the object being to utilize the heat of vaporization of the water, which vaporization is furthered by any air currents which may strike against the coils. From this point on the ammonia is practically free from water, so that the next portion of the cooling coil drains forward into the ammonia tank in which the liquid ammonia is stored at practically atmospheric temperature until it passes down through the expansion valve. This portion of the apparatus is identical in both the compressor and absorption systems, and makes use of the cooling effect occasioned by the unresisted expansion from high to low pressure. This operation produces a slight vaporization of the ammonia, and to that extent destroys its refrigerative value. The remaining liquid passes into the refrigerating coils which may be located either directly in the refrigerating rooms, or inserted in a brine solution in case indirect refrigeration is desired. After

being vaporized, the ammonia passes into the absorbent, as mentioned above. During absorption the ammonia liberates about 900 B.T.U. per pound. The resulting increased temperature of the mixture from such a generation of heat would prevent the absorption of ammonia, so that this heat of absorption must be eliminated from the generator by the circulation of cold water from suitable pipe coils. It is evident that the water which is left behind in the generator, when the ammonia is driven off by heat, must be drawn off, otherwise the generator would soon be filled, and similarly the water which is taken out of the absorber must be replaced, or else the absorber will soon empty. Both of these needs, can be fulfilled by permitting the water to return from the generator to the absorber. Naturally the weakest solution will be found at the bottom of the generator, so that the return pipe takes the water from that point. We have now two streams passing in opposite directions; a rich solution passing from the analyzer to the generator, which must be heated in order to give off the ammonia, and a weak solution passing from the generator to the absorber which must be cooled in order to be able to absorb the ammonia. Evidently, then, both of these purposes can be effected by permitting the two streams to flow in opposite directions, but in thermal contact, so that the rich solution will be warmed by taking up the heat of the weak solution. Of course this necessitates that a force pump be located between the absorber

and the point at which this heat is received by the strong solution. This device, which transfers the heat from the weak to the strong solution, is known as an "inter-changer" or "regenerator." Such a system appears complicated at first sight, but after a little thought becomes fairly simple if we simply stop to realize that the compressor is being replaced by a water conveyor, which must first be cooled and then heated in order to absorb and deliver the ammonia, and that most of the complexities of the mechanism arise in carrying out these two functions of heating and cooling, and the further function of separating the water vapor entirely from the ammonia.

There are really three separate cycles being followed throughout the mechanism. There is the cycle of the water, the cycle of the ammonia, when free and when absorbed, and finally, in case of indirect refrigeration, there is also a brine solution. These different cycles or paths may be outlined as follows: The weak solution starts at the generator, passes through the exchanger to the absorber where it meets the gaseous ammonia, and the two unite. The strong solution is then taken by the pump, forced through the exchanger, this time on the high pressure side of the pump, and separated from the hot weak solution only by the thickness of the piping, and then enters the top of the analyzer. At this point most of it falls downward toward the generator, a small portion is perhaps vaporized and passes up into the rec-

tifier, and then falls back again into the generator, so that with the exception of a few particles which keep whirling around in eddy currents between the analyzer and the rectifier, most of it passes down into the generator, and, having left all the ammonia behind, is drawn off as a weak solution and begins its cycle once more. The ammonia leaves the generator in conjunction with the steam vapor, passes up through the analyzer undergoing partial condensation and re-evaporation, but in the main passing onward to the rectifier, where it again undergoes partial condensation and escapes from the rectifier entirely free from water. It then goes through the condenser, throttle valve, expansion coils, into the absorber, where it unites with the weak solution and travels the path already described, namely, from the absorber through the pump and exchanger to the analyzer. In case of brine refrigeration, the ammonia expansion coils are placed in the brine tanks. Then the cool brine is forced through suitable piping to the refrigerating rooms where it picks up heat and returns once more to the cooling tanks.

## CHAPTER XVIII.

### DISCUSSION OF KELVIN'S WARMING ENGINE.

**The Warming-engine.**—The third possibility of the reversed cycle, viz., the utilization of the heat delivered at the upper temperature for heating, was pointed out by Lord Kelvin; the idea being that, given, say, a definite quantity of steam for heating purposes, a greater heating effect could be obtained by utilizing the steam to run an engine and compressor system and then diverting the exhaust steam of the engine and the heated fluid of the compressor to heating purposes, than by a direct application of the steam itself.

The explanation of this fact is that in the one case the availability of the heat to perform work at the high temperature is utilized, while in the other it is lost. When heat is transferred by conduction, radiation, etc., from a hot body to a colder body, the entropy of the hot body decreases and that of the cold body increases.

But when an adiabatic transfer of energy occurs by changing heat first into work and then back into heat, by alternate expansion and compression, the entropy of each part of the system remains unchanged and the temperature of the hot body is decreased while that of the cold body is raised. The action of such a system

can best be illustrated by assuming an ideal system from which all friction and conduction losses are eliminated, and comparing the relative heating effects obtained by direct heating and by using the warming engine.

*Problem.*—A pound of dry steam at 150 pounds absolute pressure, if used in a radiator and trapped out at room temperature, say  $70^{\circ}$  F., would yield 1155 B.T.U. Assuming this steam to operate upon the Rankine cycle,

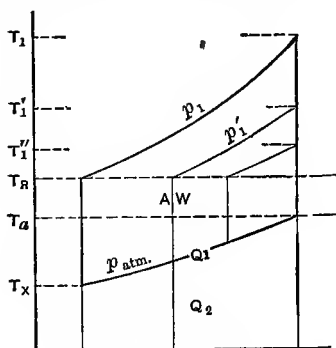


FIG. 119.

discharging non-condensing into the heating system at 17 lbs. absolute, the exhaust, before leaving the radiator as water at  $70^{\circ}$  F., would yield 1095.3 B.T.U., while 59.7 B.T.U. would be developed into work.

Let the steam-engine furnish the work required to drive a compressor-motor unit which operates with one pound of air. The compressor receives the air at atmospheric temperature and discharges it at some higher temperature  $T_1$  into radiator coils, where it is cooled under



constant pressure  $p_1$  to room temperature  $T_R$  (Fig. 119). It next passes to the motor where, after expanding to a final temperature  $T_x = \frac{T_a \cdot T_R}{T_1}$ , it exhausts into the atmosphere and a fresh charge of warmer air is taken by the compressor. The effect is equivalent to taking from the atmosphere per pound of air an amount of heat equal to  $C_p(T_a - T_x)$ , or  $0.2375T_a \cdot \frac{T_1 - T_R}{T_1}$  B.T.U. The heat rejected to the radiator equals  $0.2375(T_1 - T_R)$  B.T.U., while the work required to drive the unit is equal to  $0.2375(T_1 - T_R) \left(1 - \frac{T_a}{T_1}\right)$ . Therefore, the ratio of heat delivered to work input is  $\frac{1}{1 - \frac{T_a}{T_1}}$ , or  $\frac{T_1}{T_1 - T_a}$ .

From this it follows that the air effects a heating equal to  $\frac{T_1}{T_1 - T_a}$  times the heat developed into work. But

$$\frac{T_1}{T_1 - T_a} = \frac{C_p(T_1 - T_R)}{\Delta H},$$

$$\text{or} \quad T_1 - T_a - T_R + \frac{T_a \cdot T_R}{T_1} = \frac{\Delta H}{C_p}.$$

If we assume the temperature of the outside air to be  $0^\circ \text{F.}$  a substitution of the values  $T_a = 459.5$ ,  $T_R = 527.5$ ,  $C_p = 0.2375$  and  $\Delta H = 59.7$ , gives  $T_1 = 996.0^\circ \text{F. abs.}$  or  $t_1 = 536.5$ . From this it follows that the air rejects  $C_p(T_1 - T_R) = 0.2375(536.5 - 70) = 110.8$  B.T.U. in pass-

ing through the radiator. This represents a gain of 110.8—59.7 or 51.1 B.T.U. over the energy required to drive the compressor-motor unit, and is therefore the heat gained from the low temperature supply of the atmosphere.

In the above problem an unnecessary limitation was imposed by assuming one pound of air per pound of steam. If  $n$  pounds of air are compressed per pound of steam each pound will receive but  $\frac{1}{n}$  as much work, so that the delivery temperature will be something less than  $T_1$ , say  $T_1'$ , which may be obtained from the equation

$$T_1' - T_a - T_R + \frac{T_a \cdot T_R}{T_1'} = \frac{\Delta H}{n \cdot C_p}.$$

Assuming  $n=10$ , the delivery temperature reduces to 624.0° F. abs., or 164.5° F. The heat rejected by the air in the radiator now equals  $10 \times 0.2375 (164.5 - 70) = 224.4$  B.T.U. This represents a gain of  $224.4 - 59.7 = 164.7$  B.T.U., which is the heat obtained from the low temperature supply of the external atmosphere.

It should be noticed that although by increasing the amount of air circulated the quantity of low temperature heat which becomes available has been raised from 51.1 B.T.U. to 164.7 B.T.U, this has been obtained at the expense of a larger compressor, because less heating is done per unit weight of air as the upper temperature  $T_1$  gradually diminishes in magnitude with increasing

weight of air. Evidently the limiting condition is reached when  $T_1$  becomes equal to  $T_R$ , when there results the maximum heating effect from the air, viz.,

$$\frac{T_R}{T_R - T_a} \cdot \Delta H - \Delta H = \frac{T_a}{T_R - T_a} \cdot \Delta H.$$

In the case of the above problem this amounts to  $\frac{459.5}{70} \times 59.7 = 65.64 \times 59.7 = 391.9$  B.T.U. The combined heating effect from the steam and air is thus 1487 B.T.U. in place of the 1155 B.T.U. from the steam alone.

An investigation of the general expression for the heat gained from external sources per pound of air,  $\frac{T_a}{T_1 - T_a} \cdot \frac{\Delta H}{n}$ , indicates two methods of increasing this result. This first, which has already been mentioned, consists in making  $T_1$  approach  $T_R$  in value by increasing the quantity of air circulated. The second consists in making  $T_a$  approach  $T_R$ . This is ordinarily beyond the control of the engineer, who must take the temperature of the external air as he finds it, but it at least teaches that the smaller the increase in temperature through which the external air must be raised the greater the gain from the warming engine.

An inspection of Fig. 119 makes this evident. The lower the temperature  $T_1$  the higher the temperature  $T_x$ , since  $T_1 \cdot T_x = T_a \cdot T_R$ , and therefore the smaller the area

$AW$  relatively to the area  $Q_2$ . Thus the ratio of areas increases from  $\frac{T_x}{T_R - T_x}$  to  $\frac{T_a}{T_R - T_a}$  as the upper temperature decreases from  $T_1$  to  $T_R$ . Further, as  $T_a$  approaches  $T_R$  the value of  $\frac{T_a}{T_R - T_a}$  increases indefinitely.

**General Discussion.**—We are now in a position to give a more general discussion of this subject in that we avoid the special characteristics of any particular substance by assuming the working substances in both the engine and the compressor-motor unit to operate upon the direct and reversed Carnot cycles respectively.

Let  $T_1$  be the temperature of a limited supply of heat,  $Q_1$ , and  $T_2$  that of an unlimited supply, say, of the atmosphere, and  $T_3$  some intermediate temperature to which a room is to be warmed. A Carnot engine working between  $T_1$  and  $T_3$  would perform the work  $Q_1 \frac{T_1 - T_3}{T_1}$  and reject the heat  $Q_1 \frac{T_3}{T_1}$ . Suppose this work to be expended upon an air-compressor, heating the air from  $T_2$  to  $T_3$ . As the entropy of the air does not increase during adiabatic compression, the entropy of the heat to be taken from the air so compressed must be equal to the heat equivalent of the work of compression divided by the increase in temperature, or

$$A\phi_2 = \frac{Q_1 \frac{T_1 - T_3}{T_1}}{T_3 - T_2} = \frac{Q_1}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2},$$

hence the supply of air taken into the compressor must have available the heat

$$Q_2 - T_2 \cdot \Delta\phi_2 = Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}.$$

The heat available in the hot air at temperature  $T_3$  will then be

$$\begin{aligned} Q_A &= Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2} + Q_1 \frac{T_1 - T_3}{T_1} \\ &= Q_1 \frac{T_1 - T_3}{T_1} \left[ \frac{T_2 + T_3 - T_2}{T_3 - T_2} \right] = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}, \end{aligned}$$

which could also be obtained by multiplying the decreased entropy of the air by the final temperature  $T_3$ . Thus

$$Q_A = \Delta\phi_2 \cdot T_3 = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}.$$

The total quantity of heat delivered to the room thus becomes the sum of that rejected by both engine and compressor, or

$$\begin{aligned} Q_3 &= Q_1 \frac{T_3}{T_1} + Q_A = Q_1 \frac{T_3}{T_1} + Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2} \\ &= Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_2}{T_3 - T_2}. \end{aligned}$$

The increase in heating power over that obtained by the use of the steam alone is thus

$$Q_3 - Q_1 = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_2}{T_3 - T_2} - Q_1 = Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2} = Q_2,$$

or equal to the heat obtained from the low temperature air. An examination of this last formula shows that if the upper and lower temperatures,  $T_1$  and  $T_2$ , are fixed, the gain will be greater the smaller the value of  $T_3$ , so that when the range  $T_3 - T_2$  is small the gain may be many times the original quantity of heat. The gain  $Q_3 - Q_1$  represents the extreme difference obtained by supposing in one case all the availability to be utilized and in the other that none of it is utilized, or that the hot body simply expanded along a constant heat-curve until  $T_3$  is reached, thus suffering an increase of its own entropy. In practice the actual difference would be diminished from both sides, the maximum value of  $Q_3$  being impossible to attain, due to radiation, conduction, and friction losses, and the minimum value  $Q_1$  would always be exceeded, as the heat contained in the air would be partially utilized.

This maximum gain,  $Q_3 - Q_1$ , can be illustrated by means of the  $T\phi$ -diagram, as shown in Fig. 120. Let  $ab$  represent the quantity of heat  $Q_1$  at the temperature  $T_1$ .  $T_2$  is the temperature of the atmosphere and  $T_3$  that in the room. A Carnot engine working between  $T_1$  and  $T_3$  would perform the work  $ac$  and exhaust the heat  $db$ , the temperature of the exhaust having been lowered from  $T_1$  to  $T_3$ , but the entropy remaining constant at  $a'b$ . If no work is performed, but the hot body permitted to expand along the constant heat-curve  $H$ , the heating effect at  $T_3$  will be  $de = ab = Q_1$ . Let us rep-



To determine the position of the point  $m$ , it is convenient to construct the rectangular hyperbola  $as$  passing through  $a$ , and so proportioned that when intersected by isothermals  $T_3, T_3'$ , etc., the rectangles  $mn, m'n$ , etc., thus determined will be equal to the work performed by the Carnot engine,  $ac, ac'$ , etc., respectively. It should be noticed that as  $T_3$  approaches  $T_2$ , the quantity of heat  $nf, nf'$ , etc., utilized from the atmosphere increases indefinitely.

**A Gas as the Working Fluid.**—The above general discussion is valuable as defining a maximum standard of reference, but the Carnot cycle is not feasible in practice, and must be replaced by the Joule cycle for gases and by the Rankine cycle for saturated and superheated vapors. An illustration of the use of the Joule cycle with air has already been given in the above problem, which shows that the theoretical gains with air heating are probably not sufficient to offset the cost, bulk, maintenance and inconvenience of a compressor-motor unit installed for this purpose alone. That is, the heating problem is different from that of refrigeration. There is but one way to obtain the necessary refrigeration and the expense must be met, but in heating the direct method is the simplest and most obvious, and the warming engine must justify itself by showing a considerable thermal saving. Of course there always exists the interesting possibilities of obtaining the heating as a by-product of the refrigerating system, or of



using the same plant for cooling in summer and heating in winter. The same advantages exist here in the use of a dense air system as in the case of refrigeration.

**A Saturated Vapor as the Working Fluid.**—The same qualities which make the use of a saturated vapor, as ammonia, advantageous for refrigeration, hold good in the case of the warming engine, viz., the large heat capacity and the small volume.

Here the cooling coils of the refrigerating system would be replaced by large radiating surfaces properly distributed about the building to be heated, and if no refrigeration was desired the ammonia heating coils (evaporation coils) could be exposed to the atmosphere, or better still, if convenient, be immersed in some nearby river or ocean and thus draw heat from this limitless supply. Due to the high vapor tension of the ammonia the temperature in the radiator coils would be less than in steam coils, so that the rate of heat flow would be less than in steam heating and consequently a greater area of radiating surface would be required.

The warming unit would consist of a steam engine, running non-condensing and exhausting into steam heating coils, which operates a direct-connected ammonia (or other saturated vapor) compressor. The heating would be effected by the exhaust steam at about 220° F. and by the high-pressure ammonia at about 100° F. The low temperature heat would be

obtained from water probably at a temperature of about 40° F.

*Problem.*—Assuming that the ammonia follows the reversed Rankine cycle between the pressure limits thus defined, with the exception that cooling along the liquid line is replaced by the throttling curve, the low temperature heat picked up at 40° F. would equal  $q_{40^\circ} + r_{40^\circ} - q_{100} = 9 + 534 - 75 = 468$  B.T.U. The work

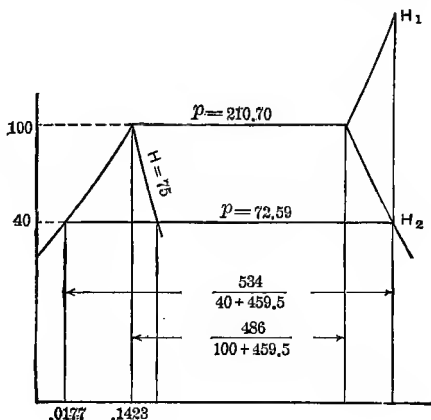


FIG. 121.

required to drive the compressor (see Fig. 121) would be  $H_1 - H_2$ . To obtain  $H_1$  we have  $\phi_1 = \phi_2$ , or

$$0.1423 + \frac{486}{559.5} + 0.54 \log_e \frac{T_1}{559.5} = 0.0177 + \frac{534}{499.5};$$

whence

$$T_1 = 643.9^\circ \text{ F. abs.} = 184.4^\circ \text{ F.}$$

Therefore

$$H_1 = 561 + 0.54 \times 84.4 = 606.6 \text{ B.T.U.,}$$

and

$$H_1 - H_2 = 606.6 - 543 = 63.6 \text{ B.T.U.,}$$

so that the net heat gained from the operation would be

$$468 - 63.6 = 394.4 \text{ B.T.U. per pound of NH}_3.$$

**Electric Heating.**—Another possibility of the warming engine is the economical utilization of electrical power for heating purposes. The direct utilization of electric power in resistance heaters is much less economical than the customary furnaces, hot water and steam heaters, but if the electric power is used to operate an electric motor-compressor unit it is possible that the efficiency of the warming engine may bring such a usage within the realm of economic possibilities.

Thus, imagine the compressor of the preceding problem driven by such a motor, then assuming no losses, the heating accomplished by the warming engine would be  $\frac{468}{63.6} = 7.36$  times that of the heat equivalent of the electrical energy. Of course this full theoretical value would not be realized in practice due to stray power and friction losses, but it should be noted that such losses are not a direct thermal loss as they possess the

same heating effect whether obtained in the motor windings and the bearing surfaces or in a resistance coil, but that the only "loss" which they signify is a limitation of the amount of increase to be obtained from the warming engine. It is possible that the net returns would be sufficient to bring the cost of electricity within competitive range with coal, especially as the greater demand for such heating current would come during the day so that the manufacturer could afford to sell the current at low rates in order to increase the load factor of the central station, and thereby lower the cost of power to all consumers and in this way help to extend the applications of electricity in other directions.

"Thermodynamically our present methods of heating are most wasteful and subject to great improvement and sooner or later as the limitations of our natural resources begin to be felt a more economical method will be developed. When, therefore," to quote the words of Prof. Cotterill, "we warm our houses by the direct action of heat from combustible bodies, we waste by far the greater part of it by making no use of the high temperature at which the heat is generated, a small quantity of heat at high temperature being ideal capable of raising a large quantity to a moderate temperature."

"It is interesting, and may some day be useful," says Prof. Ewing, "to recognize that even the mo-

economical of the usual methods employed to heat buildings, with all their advantages in respect of simplicity and absence of mechanism, are in the thermodynamic sense spendthrift modes of treating fuel."

## CHAPTER XIX.

### ENTROPY ANALYSIS IN THE BOILER ROOM.

**The Steam-boiler.**—The boiler is, thermally, the least economical member in any steam plant installed for the production of power. The hot gases passing over the heating surfaces must be of considerably higher temperature than the water in the boiler, so as to insure a rapidity of heat transference commensurate with reasonable development of power. The higher therefore the boiler pressure, other things being equal, the greater the amount of heat carried off in the exhaust gases. Practice shows that the heat lost through the setting and up the stack varies roughly from twenty per cent to fifty per cent of the heat of combustion of the fuel. The mere statement, however, that the boiler efficiency varies from sixty per cent to eighty per cent does not present the total wastefulness of the boiler when the steam is to be used in the production of power and not for mere heating and industrial purposes.

Suppose, for example, that two boilers develop thermal efficiencies of seventy per cent and sixty-five per cent while supplying steam at 100 lbs. absolute and 250 lbs. absolute, respectively, to two condensing engines exhausting at 1 lb. absolute back pressure. The second boiler

is really the more efficient thermally. The exhaust pressure is determined by the effectiveness of the condenser and the vacuum pumps and is beyond the control of the boiler, and is therefore made equal in this illustration, but the pressure during admission is determined by the boiler and the effects of its variation must be charged to the boiler alone. The temperature corresponding to the boiler pressure determines the motivity or availability of the heat absorbed in the boiler. The temperatures corresponding to 250, 100 and 1 pounds are 401.1, 327.9 and 101.8 degrees Fahrenheit, so that the motivities in the two cases cited above become

$$\frac{327.9-101.8}{327.9+459.5} \quad \text{and} \quad \frac{401.1-101.8}{401.1+459.5}$$

or 28.7 per cent and 34.8 per cent. So that the net motivity of the heat of combustion of the coal is  $.70 \times 28.7$  and  $.65 \times 34.8$ , or 20.1 and 22.6 respectively. It is thus evident that the boiler possessing the greater "boiler efficiency" may really be the less efficient when its effect upon the availability of the heat is considered.

**Loss of Availability between the Fire and the Steam.**— Besides the direct heat losses of 30 and 35 per cent quoted in the above problem there is a further loss due to drop of temperature of 49.9 and 42.4 per cent respectively. That is, the loss in efficiency due to drop in temperature between the hot gases in the furnace and

the steam in the boiler is roughly one and a half times that due to the direct heat losses.

The loss of availability of the heat in a boiler is clearly illustrated by the temperature entropy diagram, as shown in Fig. 122. Let  $abdc$  represent a certain quantity of heat  $Q$  which leaves the gases of the furnace at the temperature  $T_F$ , and let  $a'b'd'c$  represent

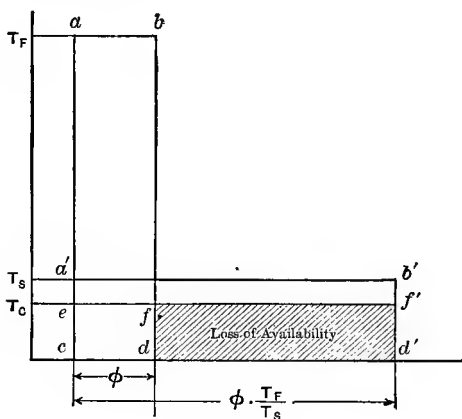


FIG. 122.

this same quantity of heat after it has entered the water of the boiler at the temperature  $T_s$ . As the temperature has decreased in the ratio  $\frac{T_s}{T_F}$  the entropy factor must have increased in the ratio  $\frac{T_F}{T_s}$ , so that if the original entropy  $cd$  possesses the magnitude  $\phi$ , the final entropy  $cd'$  must possess the entropy  $\frac{T_F}{T_s} \cdot \phi$ . Thus



while the entropy of the hot gases in the furnace has decreased by the amount  $\phi = \frac{Q}{T_F}$ , the entropy of the steam has increased by the amount  $\frac{T_F}{T_s} \cdot \phi = \frac{Q}{T_s}$ , and there has therefore resulted a net increase in the entropy of the system of the amount  $\phi \left( \frac{T_F}{T_s} - 1 \right)$ .

If now  $T_c$  represent the temperature of the condenser, i.e., the coldest temperature practically attainable, the area  $efdc$  represents the heat theoretically unavailable for work, while the heat  $Q$  is still contained in the gases, while  $ef'd'c$  represents the heat no longer available after the heat  $Q$  has entered the steam. The increase in non-availability or the loss in motivity, is thus represented by the area  $ff'd'd$ , and is equal to the increase in entropy multiplied by the lowest of available temperatures, or

$$T_c \cdot \phi \left( \frac{T_F}{T_s} - 1 \right).$$

This increase in non-availability is thus seen to be equal to the heat unavoidably lost under the original conditions,  $T_c \cdot \phi$ , multiplied by a factor  $\frac{T_F}{T_s} - 1$ , which increases in value as  $T_s$  decreases, and only disappears when  $T_s = T_F$ , i.e., when the heat is used at the temperature of the fire.

**Loss of Availability Due to the Liquid Line.**—Due to the fact that two out of the four operations constituting

the Rankine cycle occur in the boiler a further loss must be charged against it. The deviation of the Rankine cycle from the Carnot cycle has already been discussed upon pp. 233–234, and was found to be due to the slope of the liquid line as determined by the magnitude of the specific heat of the liquid used.

The full motivity of the heat as determined by the boiler pressure is not possible of attainment, as part of the heat absorbed by the water is received at temperatures less than that corresponding to this pressure. This can be remedied only by heating the feed water to boiler temperatures before it enters the boiler.

The deviation between the Carnot and Rankine cycles for any given case can be readily obtained by computing the motivity  $\frac{T_s - T_c}{T_s}$  and comparing it with the corresponding Rankine efficiency taken from the table on p. 231.

The losses due to the boiler may therefore be summarized as

- (1) Heat lost through the setting and up the stack.
- (2) Loss of availability due to reduced temperature.
- (3) Loss of availability due to use of Rankine cycle in place of the Carnot cycle.

**The Regenerative Principle.**—In the discussion of isothermal cycles on pp. 6–10 it was shown that the efficiency of such cycles was equal to that of the Carnot cycle, and it was pointed out that in place of an adiabatic

expansion there should be an extraction of heat by conduction during decrease in temperature exactly equal to the heat required during the rise-in-temperature operation. (See  $bc$  and  $da$ , Fig. 3.) The application of this principle is further illustrated on pp. 158–160 in the discussion of the Stirling and Ericsson cycles for hot air engines.

The application of this principle to the cycle of water in a power plant necessitates a modification of the

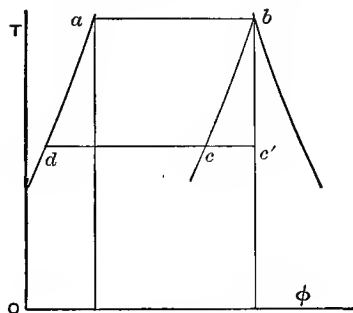


FIG. 123.

Rankine cycle whereby the adiabatic expansion line is replaced by a curve whose contour is a duplicate of the water line. That means that as the steam expands an amount of heat must be transferred from it to the feed water just equal in amount to that required to heat the water at each temperature. Thus in Fig. 123 the isentropic curve  $bc'$  is replaced by the curve  $bc$  possessing the same slope as the water line  $ad$ .

If it were possible to extract heat from the steam in

this fashion during its expansion it would then be possible to realize theoretically at least the maximum availability of the heat in the boiler.

A complete utilization of this principle is not feasible with steam engines as constructed, but in the case of compound engines it is possible to approximate it by taking sufficient steam from each receiver to warm the

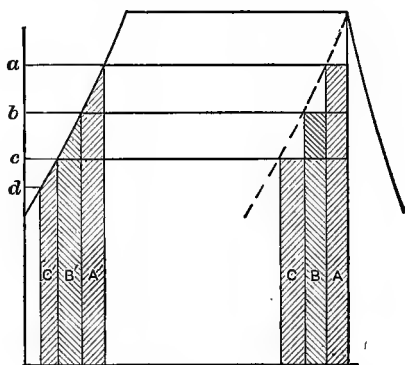


FIG. 124.

feed water up to the temperature of the receiver steam. Thus for example in a triple expansion engine operating with cold feed water, Fig. 124, the heat *A* can be taken from the first receiver and used to warm the feed water from *b* to *a*, and the heat *B* can be taken from the second receiver and serve to warm the feed water from *C* (the temperature of exhaust) to *b*, and finally the heat *C* can be taken from the exhaust steam and used to warm the feed water from *d* (the temperature in the supply pipe) to *c*.

Although this regenerative principle as thus applied may seem startling, the first time it is considered a moment's thought will show that one is already familiar with part of it under different names. Thus the quantity of heat  $C$  is saved in all economical plants with a primary heater, while part of  $B$  at least is saved by utilizing the exhaust of the auxilliary engines which ordinarily discharge at atmospheric pressure. And finally the equivalent of the rest of  $B$  and part of  $A$  is obtained by saving some of the heat of the exhaust gases with an economizer.

The use of the economizer is not truly a part of this regenerative principle, but is the utilization of heat otherwise wasted, and this opens up the interesting question as to whether it is better to use small heating surface in the main boiler combined with a good economizer and no regenerative action from the intermediate cylinders, or to make the heating surface of the boiler so extensive that nothing remains for the economizer to do, combined with regenerative action at the various intermediate receivers.\*

**The Primary Feed-water Heater.**—Ideally it would be perfectly feasible to return the condensate or an equal amount of new feed water to the boiler at the temperature of the exhaust steam; practically the temperature is always considerably less than this because

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\* *Engineering*, 1895, pp. 63, 97, 191. Feed-Water Heaters, by Professor A. C. Elliott.

of heat losses due to radiation from the feed pipe between the heater and the boiler, or between the primary heater and any secondary device which may be employed. The use of a primary heater is based upon the impossibility or rather the practical unfeasibility of reducing the temperature of the exhaust steam to that of the cooling water. That is, the small increase in power is offset by greater increased loss in the condenser plant.

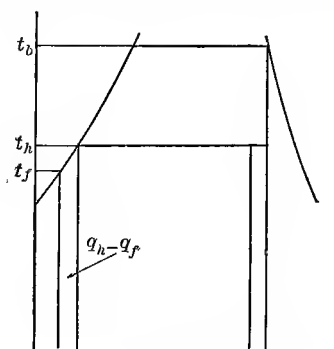


FIG. 125.

In the case of engines using vapors other than steam where the pressure at exhaust is still above atmospheric, the gain in power from the lowering of the back pressure line justifies the maintenance of a lower temperature in the exhaust condenser, so that the question of a primary feed-fluid heater is eliminated both from the thermal as well as the practical point of view. (See temperatures quoted on pp. 255-257 for the  $\text{SO}_2$  and  $\text{Æthylamine}$  engines.

The saving to be credited to the primary heater is the decrease in the amount of heat absorbed by the water in the boiler occasioned by its use. Thus if the heater raises the temperature from  $t_f$  to  $t_h$  (Fig. 125) the decrease in the amount of heat absorbed in the boiler per pound of steam is

$$q_h - q_f,$$

while the total heat absorbed per pound without the heater is

$$q_b - q_f + x r_b,$$

and therefore the efficiency of the heater is equal to

$$\frac{q_h - q_f}{q_b - q_f + x r_b}.$$

The denominator of this expression, assuming the steam to be dry and the cold feed water to be at an average temperature of 70° F., fluctuates in value from 1130 B.T.U. per lb. at 40 lbs. gage to 1165 B.T.U. per lb. at 330 lbs. gage, or undergoes a variation of less than four per cent for a temperature range of 140° F. Between 100 lbs. gage and 150 lbs. gage the variation is from 1147 to 1156 B.T.U. per lb., or less than one per cent. For rough calculations the average value of the denominator may be assumed as 1160 B.T.U. per lb.

The value of the numerator shows the effectiveness of the feed water heater, and it is evident that the efficiency is directly proportional to the increase in the

temperature of the feed water and equals approximately  $\frac{q_h - q_f}{1160}$ ; so that on an average 11.6° F. increase in the temperature of the feed water corresponds to one per cent saving in coal, and an increase of 100° F. corresponds to a saving of about nine per cent in coal.

This thermal saving is but one of many economies instituted by a feed-water heater. The heating of the feed water serves to precipitate many of the impurities in the boiler feed in the heaters, whence they may be easily removed, instead of in the less easily accessible boiler. This results in less impairment of the conductivity of the heating surface and effects a saving in fuel varying, according to the manufacturers, anywhere from fifteen per cent to fifty per cent with hard feed waters, while the average saving with soft waters is about ten per cent.

A further saving is obtained from the decreased chilling of the boiler from the introduction of cold water, thus diminishing the internal stresses due to temperature changes, reducing the repair bills upon the boiler, and prolonging its life.

**The Secondary Feed-water Heater.**—The primary heater operating with exhaust steam at one, two or three pounds pressure can raise the temperature of the feed water only to 102°, 126°, or 142° F. With condensing engines there is a steam consumption by the auxiliaries of about ten per cent that of the main



engine which is usually exhausted to the atmosphere at 212° F., or slightly in excess of this temperature. It is customary to utilize part of the waste heat in this exhaust to raise the temperature of the feed coming from the primary heater to as near 212° F. as possible.

Thus in the case of three pounds back pressure it is possible to heat the feed water from 142° F. to 212° F. or through 70° F. On the assumption of ten per cent consumption by the auxiliaries, 11 pounds of feed water must be heated for each pound used by the auxiliaries, i.e., 770 B.T.U. would need to be supplied by the secondary heater. Each pound of steam at 212° F. possesses a heat of condensation equal to 970 B.T.U., so that, assuming the exhaust from the auxiliaries to be fifteen per cent condensed, this leaves  $970 \times .85 = 823$  B.T.U. available for heating, which is more than ample. In case the main engine exhausts at lower pressure a greater heat demand is made upon the secondary heater, but this adjusts itself automatically to a considerable extent by the increased steam consumption of the auxiliaries.

The total saving due to primary and secondary heaters expressed in per cent of the heat required by the steam without such heaters is given by

$$\frac{q_{212} - q_f}{q_b - q_f + x r_b},$$

which, upon the assumption of 70° F. as the average temperature of the supply water, reduces to

$$\frac{142}{1160} = .122,$$

or 12.2 per cent at ordinary boiler pressures.

**Saving Effected by an Economizer.**—Advantage may be taken of the unavoidable loss of heat up the stack to offset, at least partially, the loss caused by cold feed water. Although the hot gases cannot be cooled below the temperature of the boiler and must in fact be considerably hotter than the steam to insure rapid flow of heat, they are still hot enough to impart considerable heat to the colder feed water. This is accomplished by passing the feed water on its way to the boiler through suitable piping installed in the path of the hot gases.

Such an economizer will not deliver the water to the boiler at boiler temperature, but will succeed in raising it far above the temperatures obtained in primary and secondary heaters. For example, tests on Green economizers show an increase in the feed temperature of about 120° F., whether the water enters the economizer at 40° F. or 200° F. making but little difference. The gases leaving the economizer possessed temperatures varying from 254° F. to 293° F.

Any such increase in temperature above that of the secondary heater represents a direct saving of heat which would otherwise be wasted. It might seem possible at first sight to bring the feed water up to the boiler temperature, but this is impracticable as a certain temperature of the exhaust gases is required to

produce the necessary draft in the chimney. If cooled beyond this point it becomes necessary to install a forced draft and then its steam cost must be charged against the saving produced by the economizer.

In many installations economizers are installed in place of secondary and sometimes even in place of primary heaters, and in such case they obtain credit for savings due to other devices, but apparently they produce about the same increase in temperature regardless of the initial temperature of the water.

PROPERTIES OF SATURATED STEAM FROM 400° F. TO  
THE CRITICAL TEMPERATURE.\*

Temperature, Degrees Fahrenheit.	Absolute Pressure in Lbs. per Sq. In.	Increase in Specific Volume Due to Vapor- ization in Cu. Ft.	Heat of the Liquid in B.T.U. per Lb.	Latent Heat in B.T.U. per Lb.	Total Heat.	Entropy of Water above 32° F.	Entropy of Vapor- ization.
$t$	$p$	$s - s$	$q$	$r$	$q + r$	$\phi_w = \theta$	$\phi_s = \frac{r}{T}$
420	308.7	1.495	390.1	818.9	1209.0	.5912	.9254
430	343.6	1.347	405.6	805.4	1211.0	.6031	.9055
440	381.5	1.215	416.1	796.2	1212.3	.6148	.8853
450	422.4	1.099	426.7	786.5	1213.2	.6266	.8648
460	466.2	.9953	437.3	777.4	1214.7	.6381	.8455
470	513.5	.9024	447.9	765.2	1213.1	.6495	.8233
480	564.4	.8194	458.5	753.5	1212.0	.6610	.8021
490	619.5	.7436	469.2	741.3	1210.5	.6723	.7808
500	678.5	.6752	479.9	728.2	1208.1	.6835	.7590
510	741.7	.6134	490.6	714.4	1205.0	.6946	.7369
520	808.5	.5565	501.4	699.8	1201.2	.7058	.7145
530	889.1	.5050	512.1	684.4	1196.5	.7166	.6917
540	956.1	.4585	523.0	668.1	1191.1	.7275	.6685
550	1036.	.4174	533.8	650.8	1184.6	.7383	.6448
560	1122.	.3798	544.7	632.4	1177.1	.7490	.6204
570	1212.	.3457	555.6	612.9	1168.5	.7597	.5955
580	1308.	.3131	566.5	592.2	1158.7	.7702	.5698
590	1409.	.2828	577.5	570.0	1147.5	.7807	.5432
600	1516.	.2547	588.5	546.3	1134.8	.7912	.5157
610	1628.	.2283	599.6	520.8	1120.4	.8015	.4871
620	1745.	.2037	610.6	493.2	1103.8	.8118	.4569
630	1867.	.1809	621.7	463.2	1084.9	.8221	.4252
640	2000.	.1610	632.9	430.3	1063.2	.8322	.3914
650	2137.	.1416	641.0	396.7	1037.7	.8422	.3549
660	2281.	.1217	655.2	352.2	1007.4	.8527	.3146
670	2431.	.1031	666.5	304.1	970.6	.8634	.2693
680	2586.	.0800	677.8	245.2	923.0	.8723	.2152
690	2748.	.0464	689.1	164.3	853.4	.8825	.1429
698	2882.	0	.....	0	.....	.....	0

\* See *Engineering* (London), Jan. 4, 1907.

## HYPERBOLIC OR NAPERIAN LOGARITHMS.

THE log of a number is the exponent of the power to which it is necessary to raise a fixed number, called the base, to produce the given number. In the common logs the base is 10 and in the Napierian logs the base,  $e$ , is 2.71828 . . . The mathematical relations between these two systems and any third are given by the equations,

$$10^{\log_{10} a} = e^{\log_e a} = b^{\log_b a} = a,$$

where  $a$  is any number and  $b$  is the third base.

Whence

$$\log_e a = \log_e 10 \cdot \log_{10} a = 2.3026 \log_{10} a$$

and

$$\log_{10} a = \log_{10} e \cdot \log_e a = 0.4343 \log_e a$$

and

$$\log_b a = \frac{1}{\log_e b} \cdot \log_e a, \quad \text{or} \quad = \frac{1}{\log_{10} b} \cdot \log_{10} a.$$

In general, the log of a number in any system equals either the reciprocal of the Napierian log of the base of that system times the Napierian log of the number, or equals the reciprocal of the common log of the base of that system times the common log of the number.

In any system the base of which is greater than 1, the logs of all numbers greater than 1 are positive and the logs of all numbers less than 1 are negative.

## NAPÉRIAN LOGARITHMS.

$e = 2.7182818$

$\log e = 0.4342945 = M.$

	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.00995	0.01980	0.02956	0.03922	0.04879	0.05827	0.06766	0.07696	0.08618
1.1	0.09531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4243	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7354	1.7370	1.7387



## LOGARITHMS.

Nat. Nos.											Proportional Parts.								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3262	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6655	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	6	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7



## LOGARITHMS—Continued.

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	4	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8516	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	4	4













